

Dt - 19.07.18

R.C.C

$M_5 = 1:5:10$

$M_{20} = 1:1.5:3$

$M_{10} = 1:3:6$

$M_{25} = 1:1:2$

$M_{15} = 1:2:4$

$M_{35} = 1:4:8$

M_{20} - means

$m \Rightarrow$ mix (cement + sand + aggregate)

20 $\rightarrow$ characteristics strength

$f_{ck} = 20 \text{ M-Pa}$
 $= 20 \text{ N/mm}^2$

M_{20}

ms $\rightarrow$ mild steel

HYSD $\rightarrow$ High yield strength deformed.

TMT $\rightarrow$ Thermo mechanically Treated bar

CTD $\rightarrow$ cold twisted bar

MS $\rightarrow$ Fe 250 = $f_y = 250$

HYSD $\rightarrow$ Fe 415, Fe 570, Fe 575 etc

MC

slab - 6mm, 8mm, 10mm

beam - 10mm, 12, 16, 20 mm

column - 12mm, 16mm, 20mm, 25mm, 30mm, 40mm

Strength of concrete

7 - 14 days $\rightarrow$ 75%

21 days $\rightarrow$ 90%

28 days $\rightarrow$ complete

DL-21.07.18

- * Concrete is strong in compression & weak in tensile.
- * Steel is strong in tensile & weak in compression.

Design

Design to plan or to make a drawing which is not yet constructed.

Reinforced:-

To make any structure stronger, in civil language reinforcement means that bar or rod or steel.

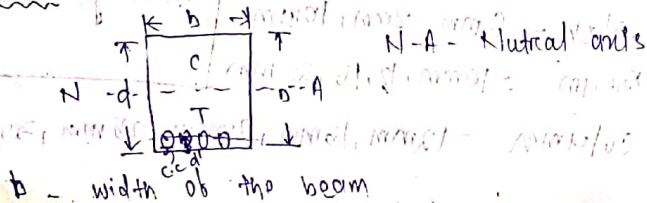
Different types of bars used in construction

6, 8, 10 mm - Slab

12, 16, 20 mm - Beam

25, 30, 40 mm - Column

Beam:-



D = over all depth

d = effective depth

d' = effective cover

$c.c$ = clear cover

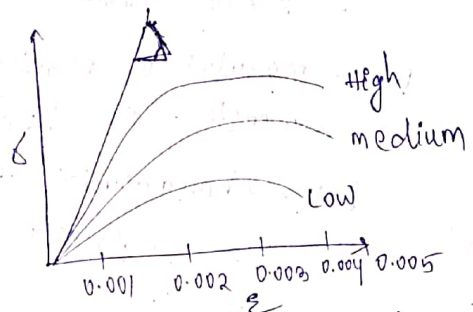
$$d = D - \frac{\phi}{2} - c.c$$

ϕ = dia. of bar

$$d' = c.c + \frac{\phi}{2}$$

$$D = d + \frac{\phi}{2} + c.c$$

Stress, strain curve



Different method of design for Reinforced cement concrete:-

- 1) Working stress method
- 2) Limit state method
- 3) Limit state strength and serviceability
- 4) Load factor method

Liberal
02190

Working stress method:-

Beam:-

→ It evolved around 1900 (century).

→ The basic concept used here is modular ratio.

→ It is the modular ratio between modulus of elasticity of steel with respect to modulus of elasticity of concrete.

$$m = \frac{E_s}{E_c}$$

where, E_s = modulus of elasticity of steel
 $= 2 \times 10^5 \text{ N/mm}^2$

E_c = modulus of elasticity of concrete
 $= 5000 \sqrt{f_{ck}}$

f_{ck} = characteristic strength

Page 46549

$$= \frac{2 \times 10^5}{5000 \sqrt{f_{ck}}} \text{ mm}$$
$$m = 22.36$$

$$m = \frac{280}{35 f_{ck}}$$

σ_{cbc} = Permissible stress

σ_{cbc} = Permissible stress concrete in

Bending moment equation

$$\frac{M}{I} = \frac{f}{y}$$

I = moment of inertia = $\frac{bd^3}{12}$

f = force

y = central distance



$$M = \frac{f \times I}{y}$$
$$= f \times \frac{b d^3}{12 \times \frac{d}{2}} = \frac{f b d^2}{6}$$

$$M = 0.167 F b d^2$$

M_{20} (Cube ^{for concrete} strength) - 20 or, compressive strength

factor of safety for concrete - 3

" " for steel - 1.8

Steel - yield strength (Tensile strength)

Assumption made for 'WSM' (working stress method)

1) Both concrete & steel acts together

2) It is perfectly elastic at all

stages of loading

- 1) All the compressive stress is taken by concrete and tensile is taken by steel.
- 2) The factor of safety for concrete is 2 and factor of safety for steel is 1.8.

Defects in "WSM":

- i) It only deals with elastic behaviour of concrete & steel.
- ii) It results in larger percentage of compression steel.
- iii) concrete does not have any definite modulus of elasticity. (depends on f_{ck})

Page - 80, B.2.1.1 (clause no.)

Page - 21, table no-21 for permissible stresses

under - good

over -

balanced

over - Bad

Permissible stresses in concrete

Grade of concrete	Permissible stress in compression	Permissible stress in bond	
		Bending	Direct
M15	5.0	5.0	4.0
M20	7.0	7.0	5.0
M25	8.5	8.5	6.0
M30	10	10	8.0

Page no. 82, Table no-21, IS 456:2000

To calculate the tensile stresses is equal to.

$$\text{Tensile stress} = \frac{F_t}{A_c + m A_{st}}$$

Here

F_t = (Total tension of the member) - (Pre tension in steel)

A_c = cross-sectional area of concrete

m = modular ratio

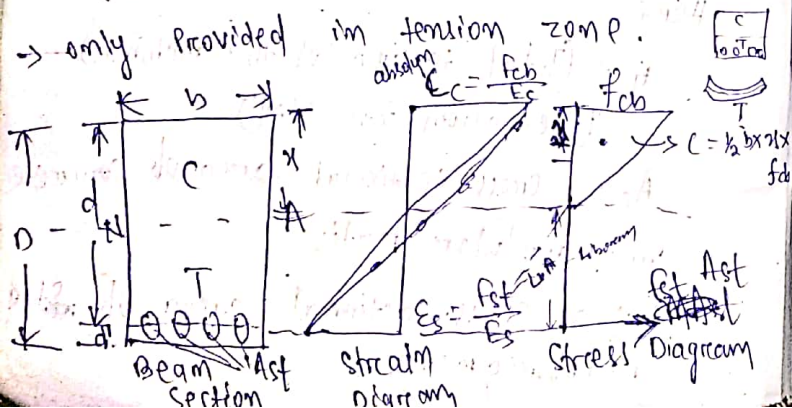
A_{st} = cross-sectional area of steel reinforcement

Singly Reinforced section beam:-

Assumption for flexure:-

- i) At any cross-section, plane section before bending remains same after bending.
- ii) All tensile strength are taken by steel and none by concrete.
- iii) The stress strength relationship of steel and concrete under working load is a straight line.
- iv) There exist a perfect bond between steel and concrete.
- v) The modular ratio 'm' is 280/56bc.

Singly Reinforced section beam:-



$$\epsilon = \frac{\sigma}{E}, \sigma \propto E, \sigma = E\epsilon$$

$$E_c = 2.5 \times 10^4$$

$$D_t = 25.07.18$$

x = Depth of neutral axis

D = overall depth

d = effective depth.

$$x = k_0 (T \cdot A)$$

$k = \text{constant}$

$$z = d - x/3$$

$$= d - \left(\frac{k_0 d}{3} \right)$$

$$= d \left(1 - \frac{k}{3} \right)$$

$$z = j d (L \cdot A) \quad (\text{L = bottom axis})$$

$$M = C \perp \text{ or } T \perp$$

$$M = \frac{1}{2} \times f_{cb} \times b \times x \times z$$

$$= \frac{1}{2} \times f_{cb} \times b \times k_0 d \times d$$

$$= \frac{1}{2} \times f_{cb} \times k \times j \times b d^2$$

$$M = Q b d^2$$

$$Q = \frac{1}{2} \times f_{cb} \times k \times j$$

Q = (moment factor)

For a given width of 250mm and 400mm over all depth. Let us consider a clear cover of 25mm and the steel area is 5-20mm. The grade of concrete is M_{20} . Find the constant and mil of steel reinforcement.

Soln

Given data:-

$$b = 250 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$C.C = 25 \text{ mm}$$

$$A_{st} = 5-20 \text{ mm}$$



We know

$$k = \frac{m f_{cb}}{f_{st} + m f_{cb}}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570 \text{ mm}^2$$

$$\left[\frac{25 \text{ mm} \times 491 \text{ mm}^2}{200} \right]$$

M_{20}

$$f_{cb} = 7 \text{ N/mm}^2$$

$$f_{st} = \frac{250}{1.8} = 138.8 \approx 140$$

$$f_{st} = 140 \text{ N/mm}^2$$

$$k = \frac{m f_{cb}}{f_{st} + m f_{cb}}$$

$$m = \frac{280}{36 f_{cb}} = \frac{280}{3 \times 7} = 13.3$$

$$k = \frac{13.33 \times 7}{140 + 7 \times 13.33} = 0.4$$

$$d = D - \frac{\phi}{2} - C.C$$

$$= 400 - \frac{20}{2} - 25 = 365 \text{ mm}$$

$$j = 1 - k \times \frac{1}{3}$$

$$j = 1 - 0.4 \times \frac{1}{3} = 0.87$$

$$j = 1 - \frac{0.4}{3} = 0.87$$

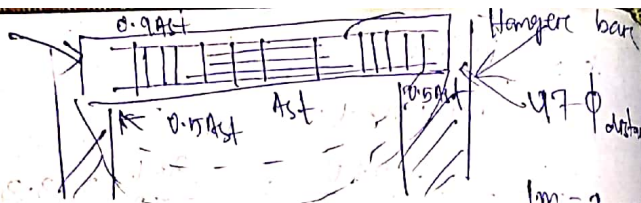
$$Q = \frac{1}{2} \times f_{cb} \times k \times j$$

$$= \frac{1}{2} \times 7 \times 0.4 \times 0.87$$

$$= 1.21$$

$$2P = \frac{f_{cb}}{f_{st}} \times k$$

$$P = \frac{7}{140} \times \frac{0.4}{2} = 0.01$$



$$l_m = 3.28$$

ϕ - dia (mm)
dia (HYSD)

$$D1 - 26.07.13$$

Grade of OPC

- 33 - N/mm²
- 43 "
- 53 "

Strength of serviceability
deflection & cracking

δ = slop

y = deflection

Fig-75, 78

clear cover
Nominal cover

Equivalent

- mild $\rightarrow$
- moderate -
- severe -
- very severe -
- extreme -

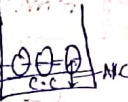
clear cover for beam - 20mm
(minimum)

Slab - 15mm (minimum)

footing - 50mm (")

column - 20mm (")

It is the clear distance betⁿ to the bottom of the beam.



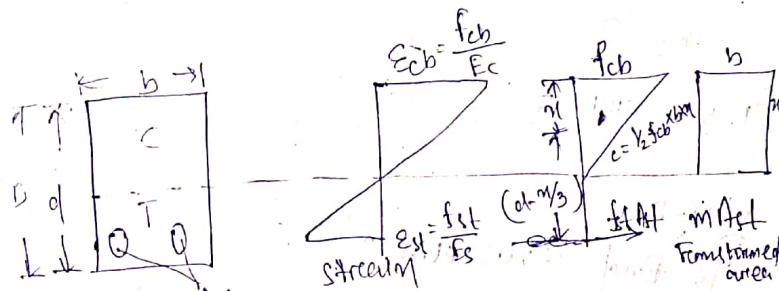
Specing:- d b

Page - No - 94

$A_{st \text{ reqd}} < A_{st \text{ bal}}$ (U.R.S)

$A_{st \text{ reqd}} > A_{st \text{ bal}}$ (O.R.S)

Assignment - Prob-1



$$\frac{f_{st}}{E_s} = \frac{f_{cb}}{E_c}$$

$$f_{st} = mcb$$

$$C = T$$

$$\text{Transformed area} \times f_{cb} = m f_{cb} A_{st}$$

Transformed area = $m A_{st}$

1 inch = 25.4 mm

To find neutral axis $x = \frac{\sum A y}{\sum A}$, $y = \frac{\sum A y}{\sum A}$
 $A_1 = b x$, $\bar{x} = \frac{x}{2}$

Here $A_1 = \text{Transformed area}$

$$A_2 = m A_{st}; x_2 = -(d - x)$$

$$\sum A x = 0$$

$$A_1 x_1 + A_2 x_2 = 0$$

$$\frac{b x^2}{2} = m A_{st} (d - x)$$

$$x = ?$$

Lever arm =

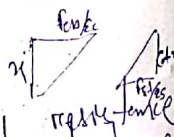
Step-2

To find lever arm?

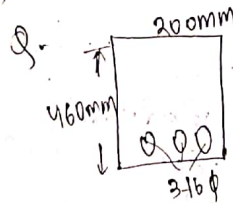
$$Z = (d - \frac{x}{2})$$

Step-3

To find the moment of ~~area~~ resistance
 $\frac{f_{cb}}{E_c} \cdot \frac{I_c}{I_{cb}} = \frac{f_{st}}{E_s} \cdot \frac{I_{st}}{I_{cb}} = \frac{f_{st}}{m} \cdot \frac{I_{st}}{I_{cb}}$ (Properties of triangle)



M.R of the section for compression
 $= \sigma_{cbc} \cdot \frac{b x}{2} \times (L \cdot A)$ (over)
 Tension $= \sigma_{st} A_{st} (L \cdot A)$ (under)



$m = 18$

Given data

$$b = 200 \text{ mm}$$

$$M = \frac{280}{3 \sigma_{cbc}} = 13.33$$

$$d = 460 \text{ mm}$$

$$A_{st} = 3 \times 201 \text{ mm}^2$$

$$= 603 \text{ mm}^2$$

$$\text{Transformed area of steel} = 13.33 \times 603 = 8038 \text{ mm}^2$$

$$\frac{b x^2}{2} - m A_{st} (d - x) = 0$$

$$\frac{200}{2} x^2 - 8038 (460 - x) = 0$$

$$100 x^2 - 3697480 + 8038 x = 0$$

$$100 x^2 + 8038 x - 3697480 = 0$$

$$\Rightarrow M = 156.2 \text{ kNm}$$

$$z = d - \frac{\gamma}{3}$$

$$= 460 - \frac{156.2}{3}$$

$$= 407.9 \text{ mm}$$

$$f_{st} = \frac{M}{A_{st}(L.A)}$$

$$= \frac{30 \times 10^6}{603 \times 407.9}$$

$$= 121.97 \text{ N/mm}^2$$

$$f_{cb} = \frac{121.97}{13.38} \times \left(\frac{156.2}{460 - 156.2} \right)$$

$$= 4.90 \text{ N/mm}^2$$

$$M = \sigma_{cbc} \times \frac{b\gamma}{2} \times (L.A)$$

$$= 7 \times \frac{200 \times 156.2}{2} \times 407.9$$

$$= 44599786 \text{ Nmm}$$

$$= 44.6 \text{ kNm}$$

$$= 44.6 \text{ kNm}$$

$$1 \text{ kNm} = 10^3 \times 10^3 \text{ Nmm}$$

$$1 \text{ kN} = 10^3 \text{ N}$$

$$1 \text{ m} = 10^3 \text{ mm}$$

11/1 - 28.07.18

$x_{act} < x_{bal}$ or x_{bal} (under reinforced section)

$x_{act} > x_{bal}$ (over reinforced section)

$x_{act} = x_{bal}$ (Balanced section)

Q. A R.C.C beam 300mm wide and 460mm effective depth is reinforced with four number (4-12mm) diameter bar in tension. Find the depth of neutral axis and state the type of beam the materials are M20 grade concrete and HYSD reinforcement of grade Fe415.

Soln

Given data:-

$$b = 300 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$A_{st} = 4-12 \text{ mm}$$

$$= 4 \times \frac{\pi}{4} \times 12^2$$

$$= 452.38$$

$$M_{20} - \sigma_{cbc} = 7 \text{ N/mm}^2$$

Fe415

$$f_{st} = \frac{415}{1.8} = 230.5$$

$$m = \frac{280}{36 \text{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$\text{Transformed area of steel} = 13.33 \times 452.3 = 6029.15 \text{ mm}^2$$

Neutral axis:- actual depth of neutral axis

$$\frac{bx^2}{2} - m A_{st} (d - x) = 0$$

$$\frac{360}{2} x^2 - 13.33 \times 452.3 (460 - x) = 0$$

$$\frac{360}{2} x^2 - 6029.15 \times 460 - 6029.15 x = 0$$

$$180x^2 - 2773409 + 6029.15x = 0$$

$$x = 197.3 \text{ mm}$$

Depth of the critical neutral axis $x_{bal} = x_c$

$$= 0.29 \times 460$$

$$= 133.4$$

~~Actual~~ x_{act} (under reinforcement section)

$$x_{act} < x_{bal}$$

$$b = 250 \text{ mm}$$

$$d = 360 \text{ mm}$$

$$A_{st} = 4-16 \text{ mm}$$

f_{e250}

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2$$

$$= 804$$

$$m_{20} \cdot 6 \text{cbc} = 7 \text{ N/mm}^2$$

$$f_{e250} = \frac{250}{1.8} = 140 \text{ N/mm}^2$$

$$m = \frac{280}{36 \text{cbc}} = \frac{280}{3 \times 7} = 13.33$$

actual depth of neutral axis

$$\frac{bx^2}{2} - m A_{st} (d - x) = 0$$

$$\frac{250}{2} x^2 - 13.33 \times 804 (360 - x) = 0$$

$$125x^2 - 13.33 \times 804 (360 - x) = 0$$

$$125x^2 - 10917 (360 - x) = 0$$

$$125x^2 - 3858120 + 10917x = 0$$

$$125x^2 + 10917x - 3858120 = 0$$

M.R = $6cbcr \frac{bx}{2} (L.A)$ Section for compressor

= ~~$6cbcr \frac{bx}{2}$~~

= $6cbcr \frac{bx}{2} \left(d - \frac{x}{3}\right)$

= $8.5 \times \frac{280 \times 159}{2} \times \left(560 - \frac{159}{3}\right)$

= ~~95929470~~

= ~~9646 kNm~~

= ~~96 x 10⁶ kNm~~

96 kNm

M.R of the section Tension = $6st Ast (L.A)$
= $6st Ast (L.A)$

= $306 \times 804 \times \left(560 - \frac{159}{3}\right)$

= 124984168

= 124. kNm

Assignment page-120
Book 5.10 - 5.17

Dt- 21.07.18

Given data:-

b = 250 mm

d = 520 mm

$6cbcr = 8.5 / \text{mm}^2$

$f_{e280.6st} = \frac{280}{1.8} = 140 \text{ N/mm}^2$

max grade, m = $\frac{280}{3 \times 8.5} = 11 \text{ N/mm}^2$

$Ast \text{ (3-20\#)}$

$3 \times \frac{\pi}{4} \times 20^2 = 942 \text{ mm}^2$

$k = \frac{11 \times 8.5}{140 + 11 \times 8.5} = 0.35$

Actual depth of neutral axis

$\frac{bx^2}{2} - m Ast (d-x) = 0$

$\frac{250}{2} x^2 - 11 \times 942 (520 - x) = 0$

$125x^2 - 3714480 + 6633x = 0$

$125x^2 + 6633x - 3714480 = 0$

$x = \frac{-6633 \pm \sqrt{6633^2 + 4 \times 125 \times 3714480}}{2 \times 125}$

$x = 191.68 \text{ mm}$

~~153.18 mm~~

Depth of critical neutral axis

$$x_{bal} = k d$$

$$= 0.35 \times 500$$

$$= 175 \text{ mm}$$

$$= 0.4 \times 500 = 200 \text{ mm}$$

$x_{act} < x_{bal}$

The beam is under reinforced

$$M.R = \frac{1}{2} \sigma_{cbc} \times b \times x \left(d - \frac{x}{3} \right)$$

$$= \frac{1}{2} \times 8.5 \times 250 \times 147.68 \left(500 - \frac{147.68}{3} \right)$$

$$= 62.76 \text{ kNm}$$

$$= 71.16 \text{ kNm}$$

M.R. is less than

$$M.R = \sigma_{st} \cdot A_{st} (L.A)$$

$$= 40 \text{ kNm}$$

Q. find the moment of resistance of the beam 230 mm wide and 460 mm effective depth there is 3 no. 20 mm dia bars the material are M20 grade concrete Fe 415 HYSD reinforcement and also state the type of beam.

Q.2.

find the moment of resistance of the beam section width 230 mm and 500 effective depth and also find the type of beam.

Q.1 Ans:

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 =$$

$$= 942 \text{ mm}^2$$

$$f_{e415}, \sigma_{st} = \frac{415}{1.8} = 230.7 \text{ N/mm}^2$$

$$m = \frac{230}{3 \times 7} = 13.33 \text{ N/mm}^2$$

$$K = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}} = \frac{13.33 \times 7}{230 + 13.33 \times 7}$$

$$= 0.29$$

Actual depth of neutral axis

$$\frac{bx^2}{2} - m A_{st} (d - x) = 0$$

$$\frac{230}{2} x^2 - 13.33 \times 942 (460 - x) = 0$$

$$\Rightarrow 115x^2 - 5776.55 + 12556.1x = 0$$

$$x = 176 \text{ mm}$$

Depth of cric N.A

$$x_{bal} = K d$$

$$= 0.29 \times (460 - \frac{176}{3})$$

$$x_{cric} = 116$$

$$x_{act} > x_{bal}$$

$$M.R = \frac{1}{2} \sigma_{cbc} \times b \times x (d - \frac{x}{3})$$

$$= \frac{1}{2} \times 7 \times 230 \times 176 (460 - \frac{176}{3})$$

$$= 56.86 \text{ kNm}$$

$$\begin{aligned} M.R &= \sigma_{st} \cdot A_{st} (d - \frac{x}{3}) \\ &= 230 \times 942 (460 - \frac{176}{3}) \\ &= 86.95 \text{ kNm} \end{aligned}$$

2)

$$b = 350 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$A_{st} = 3 \times 314 \times 20^2$$

$$= 942 \text{ mm}^2$$

$$f_{e415} ; \sigma_{st} = \frac{415}{1.8} = 230 \text{ N/mm}^2$$

$$m = \frac{230}{3 \times 7} = 13.33 \text{ N/mm}^2$$

$$K = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}} = \frac{13.33 \times 7}{230 + 13.33 \times 7} = 0.29$$

Actual depth of N.A

$$\frac{bx^2}{2} - m A_{st} (d - x) = 0$$

$$\frac{350}{2} x^2 - 13.33 \times 942 (500 - x) = 0$$

$$x = 179.6 \text{ mm}$$

Depth of critical N.A

$$104 - 1.08 \cdot 18$$

Self-weight = 25.50 kNm

Q. 5.15

Given data

$$b = 230 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$\text{Span (l)} = 4 \text{ m}$$

$$A_{st} = 4 - 12 \#$$

$$= 4 \times \frac{\pi}{4} (12)^2 = 452 \text{ mm}^2$$

$$G_{cbc} = \frac{20}{3} \text{ N/mm}^2 = 6.67 \text{ N/mm}^2$$

$$f_{e415}, G_{st} = \frac{415}{1.8} = 230 \text{ N/mm}^2$$

depth of actual depth of N.A

$$\frac{bx^2}{2} = m A_{st} (d - x)$$

$$\Rightarrow \frac{230}{2} x^2 = 13.33 \times 452 (564 - x)$$

$$x = 149.7 \text{ mm}$$

depth of critical N.A

$$= k d$$

$$= 0.29 \times 564$$

$$= 163.6 \text{ mm}$$

$$x_{act} < x_{crit}$$

The beam is under reinforcement of steel is

$$M.R = A_{st} \cdot G_{st} \left(d - \frac{x}{3} \right)$$

$$= 452 \times 230 \left(564 - \frac{149.7}{3} \right)$$

$$= 53.52 \text{ kNm}$$

$$M = \frac{wl^2}{8} (U.O.L) =$$

$$wl^2 = m \times 8$$

$$w = \frac{m \times 8}{l^2} = \frac{54 \times 8}{4 \times 4} \text{ kN/m}$$

$$= 27 \text{ kN/m}$$

Self-weight = ~~23.5~~ kN/m

~~(23.5 = 25 \times 0.23 \times 0.6 \times 25)~~

$= 25 \times 0.23 \times 0.6$

$= 3.45 \text{ kN/m}$

Addition self-w.d.l on beam

$29 = 3.45 = 23.55 \text{ kN/m}$

Analysis Prob

- Given, b, d, f_{st}
- Find 'm' using above data
- Type of beam it can be on (VRB or ORB)

Design Prob

- Given data m
- Find b, d, f_{st}
- It is only design for VRB

Dt = 04.08.18

Q. A cantilever beam of span 2.5m carries a u.d.l of 20 kN/m inclusive of its self weight design the beam for flexure as it is reinforced in tension only. The width of the beam is half the overall depth. The materials are m_{20} grade concrete & Fe415 HYSD reinforcement.

Given data:-

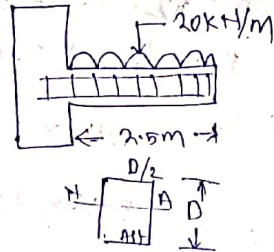
$L = 2.5 \text{ m}$

$w = 20 \text{ kN/m}$

$f_{cbc} = 7 \text{ N/mm}^2$

$f_{st} = 230 \text{ N/mm}^2$

$m = \frac{wL^2}{2} = \frac{20 \times (2.5)^2}{2} = 62.5 \text{ kNm}$



$m = Q_{bal} \cdot b \cdot d^2$

$Q_{bal} = Q_{bal} \left(\frac{d}{2} \times d^2 \right) = Q_{bal} \left(\frac{d^3}{2} \right)$

$d^3 = \frac{2m}{Q_{bal}}$

$Q_{bal} = 0.91$

$k = 0.29$

$j = 0.87$

$d_{bal} = \sqrt[3]{\frac{2 \times 62.5 \times 10^6}{0.29}}$

$d_{req} = 515.6$
 $= 515.6 \approx 516$

$d_{prov} = 1 + \frac{5}{100} \times 516$

$d_{prov} = 1.05 \times 516 = 541.8$
effective depth

$$\phi = 12, c.c = 30$$

$$D = d_{prov} + c.c + \frac{\phi}{2}$$

$$= 541.8 + 30 + \frac{12}{2}$$

$$= 541.8 + 30 + \frac{12}{2}$$

$$= 541.8 + 30 + 6$$

$$= 577.8 \approx 580$$

$$b = 290 \text{ mm}, d = 580 - 30 - 6 = 544$$

$$M_{bal} = A_{st} \cdot \sigma_{st} \cdot j \cdot d \text{ or } \left[\sigma_{st} - A_{st} \left(d - \frac{1}{3} \right) \right] \times \text{here unknown one two}$$

$$A_{st_{bal}} = \frac{M_{bal}}{\sigma_{st} \cdot j \cdot d}$$

$$= \frac{62.5 \times 10^6 \text{ Nmm}}{230 \times 0.9 \times 544 \text{ mm}}$$

$$= 55.5 \text{ mm}^2$$

$$A_{st} = n \cdot \frac{\pi}{4} \phi^2$$

$$n = \frac{A_{st} \times 4}{\pi \times \phi^2}$$

$$\Rightarrow n = \frac{55.5 \times 4}{\pi \times 144} = 4.90 \approx 5$$

$$A_{st_{prov}} = n \cdot \frac{\pi}{4} \phi^2$$

$$= 5 \times \frac{\pi}{4} (12)^2$$

$$= 5 \times \frac{\pi}{4} (144)$$

$$= 565 \text{ mm}^2$$

$$P_t \text{ (Percentage of steel)}$$

$$= \frac{100 A_{st}}{b \cdot d}$$

$$= \frac{100 \times 565}{290 \times 544} = 0.358$$

$$P_{t_{req}} = 0.358$$

$$P_{t_{bal}} = 0.44$$

$$P_{t_{req}} < P_{t_{bal}}$$

$$0.358 < 0.44$$

The section is under reinforced

Q. A simply supported beam of 6m span carries a U.d.l of 12 kN/m inclusive of its self weight, the beam is 230mm wide and effective depth is 580mm. Find the steel area. The materials are M20 grade concrete and HYSD Fe 415.

Self given data

$$l = 6m$$

$$W = 12 kN/m$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$b = 230 \text{ mm}$$

$$d = 580 \text{ mm}$$

$$m = \frac{Wl^2}{8}$$

$$= \frac{12 \times 6^2}{8}$$

$$= \frac{12 \times 36}{8} = 54 \text{ kNm}$$

$$M_{bal} = Q_{bal} b d^2$$

$$= 0.91 \times 230 \times (580)^2$$

$$= 70.4 \text{ kNm}$$

$$M_{bal} > M$$

The section is U.R.B

$$d_{prov} = \sqrt{\frac{m}{Q_{bal} \times b}}$$

$$= \sqrt{\frac{54 \times 10^6}{0.91 \times 230}}$$

$$= 507.9 \text{ mm}$$

$$507.9 \text{ mm} < 580 \text{ (U.R)}$$

$$M.R \text{ for steel} = \sigma_{st} A_{st} (d - \frac{\eta}{3})$$

$$= 230 \times A_{st} (30 - \frac{\eta}{3})$$

$$\frac{54 \times 10^6 \times 3}{230} = A_{st} (30 - \eta)$$

$$A_{st} = \frac{8.36 \eta^2}{(30 - \eta)}$$

$$704348 = A_{st} (30 - \eta)$$

$$\Rightarrow 704348 = \frac{8.63 \eta^2}{(30 - \eta)}$$

$$\Rightarrow 704348 (30 - \eta) = 8.63 \eta^2 \times 30 - 8.63 \eta^3$$

$$\Rightarrow 8.63 \eta^3 - 8.6 \eta^2 \times 30 - 704348 \eta + 704348 \times 30 = 0$$

$$\eta = 148.74 \text{ mm}$$

$$A_{st} = \frac{8.63 \times (148.74)^2}{(580 - 148.74)} = 442 \text{ mm}^2$$

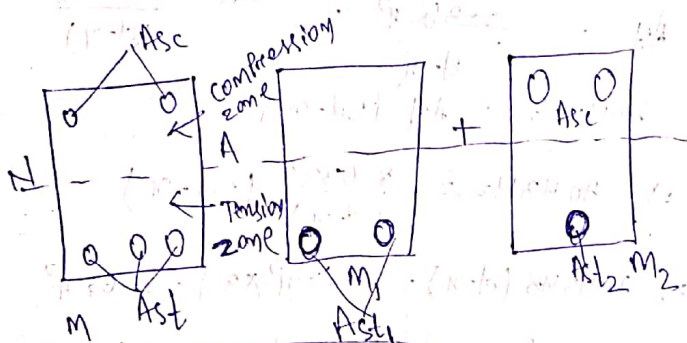
Doubly reinforced beam :-

If the design moment is more than the required moment there are two methods to design such beams.

- 1) Increase the concrete mix to increase its capacity
- 2) Provide compression reinforcement to give additional support or strength to the concrete of such beam are known as doubly reinforced beam,

DT-07.08.18

8284-Buzz Khalifa height



$$A_{st} = A_{st1} + A_{st2}$$

$$M = M_1 + M_2$$

$$M_1 = \sigma_{st} \cdot A_{st1} \cdot j \cdot d$$

The permissible stress of a compression bar in a beam or slab the compression resistance of the concrete shall be taken as the calculated compressive stress multiplied by 1.5 times of the modular ratio or σ_{cbc} .

so the transformation zone in terms of concrete area

$$1.5m \cdot A_{sc} + b \cdot x = A_{sc}$$

$$= (1.5m - 1) A_{sc} + b \cdot x$$

stress in concrete in terms of steel in compression zone

$$= \frac{x - d'}{x} \sigma_{cbc}$$

therefore the moment of resistance

$$M.R = (1.5m - 1) A_{sc} \left(\frac{x - d'}{x} \right) \sigma_{cbc} (d - d') + b \cdot x \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3} \right)$$

$$MR = (1.5m - 1) A_{sc} \left(\frac{x - d'}{x} \right) \sigma_{cbc} (d - d') + b \cdot x \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x}{3} \right)$$

$$\text{Here } M_1 = \sigma_{bal} \cdot b \cdot d^2$$

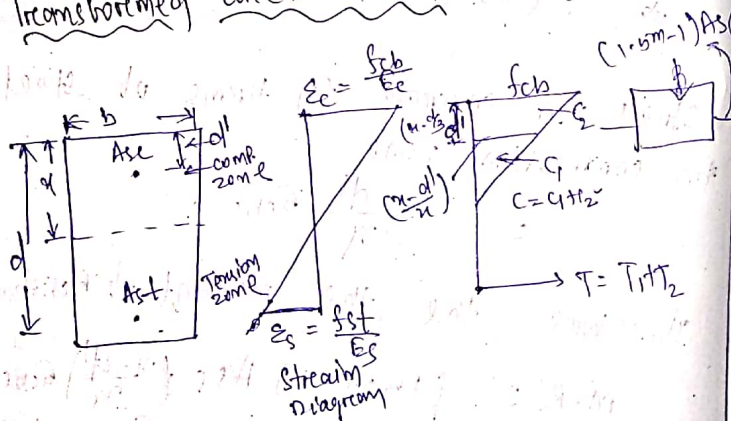
$$M_2 = (1.5m - 1) A_{sc} \cdot \sigma_{cbc} \left(\frac{x - d'}{x} \right) (d - d')$$

$$A_{st1} = \frac{M_1}{\sigma_{st} j d}$$

$$A_{st2} = \frac{M_2}{\sigma_{st} (d-d')}$$

$$A_{sc} = \frac{M_2}{(1.5m-1) \sigma_{sc} \left(\frac{x-d'}{x}\right) (d-d')}$$

Transformed area method



Transformed area of compression zone

$$= (1.5m-1) A_{sc} + b x a$$

Transformed area of Tension zone

$$= (1.5m-1) A_{st} + b x a$$

To find the position or depth of neutral axis:-

$$\frac{b x^2}{2} + (1.5m-1) A_{sc} \cdot (x-d') = m A_{st} (d-x)$$

$$\text{Moment (M)} = (1.5m-1) A_{sc} \cdot \left(\frac{x-d'}{x}\right) f_{cb} (d-x) + b x \cdot \frac{f_{cb}}{2} (d - \frac{x}{3})$$

Type of Problem:-

1. To find the depth of neutral axis and specify the type of beam.

$$x_{bal} = k d \text{ or } b x \cdot \frac{x}{2} + (1.5m-1) A_{sc} (x-d') = m A_{st} (d-x)$$

$$b x \cdot \frac{x}{2} + (1.5m-1) A_{sc} (x-d') = m A_{st} (d-x)$$

$$x_{actual} > x_{bal} \text{ (Over reinforced section)}$$

$$x_{actual} < x_{bal} \text{ (Under reinforced section)}$$

$$x_{actual} = x_{bal} \text{ (Balanced section)}$$

Step-2

To find the moment of resistance

$$M = M_1 + M_2$$

$$C_1 = \frac{bx}{2} \times f_{cb} \rightarrow y_1 = \frac{x}{3}$$

$$C_2 = (1.5m-1) A_{sc} \left(\frac{x-d'}{2} \right) \times f_{cb}$$

Here $y_2 = d'$

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

Q. A rectangular beam 230mm wide and 400mm effective depth is subjected to a moment 42 kNm. The effective cover of compressive reinforcement is 40mm. Find the reinforcing steel. The materials are M20 grade and HYSD Fe415 steel.

Given data:-

$$b = 230 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$M = 42 \text{ kNm}$$

$$d' = 40 \text{ mm}$$

$$M_{20} = A_{st}$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$A_{sc} = ?$$

$$A_{st} = A_{st1} + A_{st2} = ?$$

$$M = \frac{280}{36 \sigma_{cbc}} = 13.33$$

$$k = \frac{M \sigma_{cbc}}{\sigma_{st} + M \sigma_{cbc}} = \frac{13.33 \times 7}{230 + 13.33 \times 7} = 0.15$$

$$M_1 = Q_{bal} b d^2$$

$$= 0.91 \times 230 \times 400^2$$

$$= 33488000$$

$$= 33.5 \text{ kNm}$$

$$M_2 = M - M_1$$

$$= 42 - 33.5$$

$$= 8.5 \text{ kNm}$$

$$A_{st1} = \frac{M_1}{\sigma_{st} j d}$$

$$= \frac{33.5 \times 10^6}{230 \times 0.90 \times 400} = 402 \text{ mm}^2$$

$$A_{st2} = \frac{M_2}{\sigma_{st} (d-d')}$$

$$= \frac{8.5 \times 10^6}{230 (400-40)} = 102.62 \approx 103 \text{ mm}^2$$

$$A_{sc} = \frac{M_2}{(1.5m-1) \sigma_{cbc} \left(\frac{x-d'}{2} \right) (d-d')}$$

$$= \frac{8.5 \times 10^6}{(1.5 \times 13.33 - 1) \times \left(\frac{116-40}{2} \right) \times 360} = 291 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 508$$

To find the depth of neutral axis

$$\frac{bx^2}{2} + (1.5m-1) A_{sc} \left(\frac{x-d'}{2} \right) = M A_{st} (d-x)$$

$$\Rightarrow \frac{230 x^2}{2} + (1.5 \times 13.33 - 1) 402 \left(\frac{x-40}{2} \right) = 13.33 \times 515 (400-x)$$

$$\Rightarrow 115x^2 + 7636 (x-40) - 68605 (400-x) = 0$$

$$115x^2 + 7636x - 305440 + 68657 - 2746000$$

$$\Rightarrow x = 111.6 \text{ mm}$$

$$x_{act} < x_{crit} \text{ (VRB)}$$

Provide 2-10# & 1-12#

$$A_{st2} = 1 \times f_y \times 12^2 = 113$$

$$A_{st2} = 113 + 402 = 515 \text{ mm}^2$$

$$A_{st1} = 2 \times f_y \times 16^2 = 402 \text{ mm}^2$$

$$Q. b = 250 \text{ mm} \quad f_{ck} = 8 \text{ N/mm}^2 \quad f_{yk} = 8.5 \text{ N/mm}^2$$

$$d = 450 \text{ mm} \quad f_{yk} = 8.5$$

$$d' = 50 \text{ mm}$$

$$M = 72 \text{ kNm}$$

$$A_{st} = A_{st1} + A_{st2} = ?$$

$$A_{sc} = ?$$

$$x_{act} = ?$$

$$\sigma_{st} = 0.55 \times f_y = 0.55 \times 500 = 275 \text{ N/mm}^2$$

$$m = \frac{280}{36 \sigma_{st}} = \frac{280}{3 \times 275} = 11$$

$$k = \frac{m \sigma_{st}}{\sigma_{st} + m \sigma_{st}}$$

$$= \frac{11 \times 8.5}{275 + 8.5 \times 11} = 0.25$$

$$M_1 = Q_{bal} b d^2$$

$$= 0.91 \times 250 \times 450^2$$

$$= 48.60 \text{ kNm}$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.25}{3}$$

$$j = 0.91$$

$$M_2 = M - M_1$$

$$= 72 - 48.6$$

$$= 23.40 \text{ kNm}$$

$$Q_{bal} = \frac{1}{2} \times k \times j \times f_{ck}$$

$$= \frac{1}{2} \times 0.25 \times 0.91 \times 8$$

$$= 0.91$$

$$A_{st1} = \frac{M_1}{\sigma_{st} j d}$$

$$= \frac{48.6 \times 10^6}{275 \times 0.91 \times 450} = 431.5 \text{ mm}^2$$

$$= 432 \text{ mm}^2$$

$$A_{st2} = \frac{M_2}{\sigma_{st} (d - d')}$$

$$= \frac{23.40}{275 (450 - 50)} = 212.7 \text{ mm}^2$$

$$= 213 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2}$$

$$= 431.5 + 212.7 = 644.2 \text{ mm}^2$$

$$A_{sc} = \frac{m_2}{(1.5m - 1) \sigma_{st} \left(\frac{x - d'}{x} \right) (d - d')} - 1125$$

$$= \frac{23.40}{15.5 \times 8.5 \times 112.05 \times 400} = 799.24$$

$$= \frac{23.40 \times 10^6}{\left\{ (1.5 \times 11) - 1 \right\} 8.5 \times \left(\frac{112.5 - 50}{112.5} \right) (450 - 50) \}$$

$$= \frac{23.40}{15.5 \times 8.5 \times 112.05 \times 400}$$

$$= 799.24$$

Provided Assume dia. of rod (d) = 16 mm & 12 mm

$$A_{st1} = \frac{\pi}{4} \times d^2$$

$$\Rightarrow D_1 = \frac{A_{st1} \times 4}{\pi \times d^2}$$

$$\Rightarrow = \frac{432 \times 4}{\pi \times 16^2} = 2.14 = 3$$

$$D_2 = \frac{A_{st2} \times 4}{\pi \times d^2}$$

$$= \frac{213 \times 4}{\pi \times 12^2} = 1.8 = 2$$

$$D = \frac{799.24}{\pi/4 \times 16^2} = 3.99 = 4$$

$$A_{sc \text{ pro}} = \pi \times 4 \times \pi/4 \times 16^2 = 804 \text{ mm}^2$$

$$A_{st1} = 3 \times \pi/4 \times 16^2 = 603.18$$

$$A_{st2} = 2 \times \pi/4 \times 12^2 = 226.19$$

$$A_{st} = 829.37 \text{ mm}^2$$

Find the depth of neutral axis

$$\frac{bx^2}{2} (1.5m-1) A_{sc} (x-d') = m A_{st} (d-x)$$

$$\Rightarrow \frac{250}{2} x^2 + \left\{ (1.5 \times 11) - 1 \right\} 804 (x-50) - 11 \times 82937 (450-x)$$

$$\Rightarrow 125x^2 + 12467x - 623100 + 9123.07x - 4105381.5$$

$$\Rightarrow x = 126.5 \text{ mm}$$

8.1

$$b = 250 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$d' = 50 \text{ mm}$$

$$m = 60 \text{ kN/m}$$

$$m_{25} = \sigma_{cbc} = 8.5 \text{ N/mm}^2$$

$$f_e \text{ 500}$$

$$\sigma_{st} = 0.55 f_y = 0.55 \times 500 = 275 \text{ N/mm}^2$$

$$A_{st} = ?$$

$$A_{sc} = ?$$

$$x_{act} = ?$$

$$M_1 = Q_{bal} b d^2$$

$$= 0.91 \times 250 \times 450^2$$

$$= 48.6 \text{ kNm}$$

$$m = \frac{280}{36 \sigma_{cbc}} = 11$$

$$k = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}}$$

$$= 0.25$$

$$j = 1 - k/3 = 0.91$$

$$A_{st1} = \frac{M_1}{\sigma_{st} j d}$$

$$= \frac{48.6 \times 10^6}{275 \times 0.91 \times 450}$$

$$= 431.5 = 432 \text{ mm}^2$$

$$Q_{bal} = \frac{1}{2} \times k \times j \times \sigma_{cbc}$$

$$= 0.96$$

$$\begin{aligned}
 m_2 &= m - m_1 \\
 &= 60 - 48.6 \\
 &= 11.4 \text{ kNm}
 \end{aligned}$$

$$\begin{aligned}
 \chi &= kd \\
 &= 0.25 \times 450 \\
 &= 112.5
 \end{aligned}$$

$$\begin{aligned}
 A_{st2} &= \frac{m_2}{\sigma_{st} (d - d')} \\
 &= \frac{11.4 \times 10^6}{275 \times (450 - 50)} = 103.6 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{sc} &= \frac{m_2}{(1.5m - 1) \sigma_{cbc} \left(\frac{\eta - d'}{\eta} \right) (d - d')} \\
 &= \frac{11.4 \times 10^6}{(1.5 \times 11 - 1) 8.5 \left(\frac{112.5 - 50}{112.5} \right) 450} \\
 &= 389.3
 \end{aligned}$$

provide dia. of rod 16mm & 12mm

$$A_{st1} = D_1 \pi/4 d^2$$

$$D_1 = \frac{A_{st1} \times 4}{\pi \times d^2}$$

$$D_1 = \frac{432 \times 4}{\pi \times 16^2} = 2.14 = 3$$

$$D_2 = \frac{A_{st2} \times 4}{\pi \times d^2}$$

$$= \frac{103.6 \times 4}{\pi \times 12^2} = 0.91 \approx 1$$

$$\begin{aligned}
 \eta &= \frac{A_{sc}}{\pi/4 \times 16^2} \\
 &= \frac{389.3}{\pi/4 \times 16^2} = 1.93 \approx 2
 \end{aligned}$$

$$\begin{aligned}
 A_{sc \text{ prov}} &= \eta \times \pi/4 \times d^2 \\
 &= 2 \times \pi/4 \times 16^2 \\
 &= 402 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{st1 \text{ prov}} &= 3 \times \pi/4 \times 16^2 \\
 &= 603 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{st2 \text{ prov}} &= 1 \times \pi/4 \times 12^2 \\
 &= 113 \text{ mm}^2
 \end{aligned}$$

$$A_{st} = 113 + 603 = 716 \text{ mm}^2$$

Find the actual depth of neutral axis

$$\frac{bx^2}{2} + (1.5m - 1) A_{sc} (\eta - d') = m A_{st} (d - \eta)$$

$$\Rightarrow \frac{250x^2}{2} + (1.5 \times 11 - 1) \times 402 (\eta - 50) = 11 \times 716 (450 - \eta)$$

$$\Rightarrow 125x^2 + 6231\eta - 311550 + 7876\eta - 354420 = 0$$

$$\Rightarrow 125x^2 + 6231\eta - 311550 + 7876\eta - 354420 = 0$$

$$\Rightarrow \eta = 128 \text{ mm}$$

$$\eta_{act} < \eta_{cri} \text{ (O.K.)}$$

② $b = 230 \text{ mm}$ $\sigma_{st} = 230 \text{ N/mm}^2$
 $d = 400 \text{ mm}$ $\sigma_{cbc} = 7 \text{ N/mm}^2$
 $M = 60 \text{ kNm}$ $A_{sc} = ?$
 $d' = 40 \text{ mm}$ $A_{st} = ?$
 $m_{20} = A_{st}$ $\alpha_{act} = ?$
 Fe 415

$m_1 = Q_{bal} b d^2$
 $= 0.91 \times 230 \times 400^2$
 $= 33.5 \text{ kNm}$

$M_2 = M - m_1$
 $= 60 - 33.5$
 $= 26.5 \text{ kNm}$

$A_{st1} = \frac{m_1}{\sigma_{st} \cdot d} = \frac{33.5 \times 10^6}{230 \times 0.91 \times 400}$
 $\Rightarrow A_{st1} = 404.6 \text{ mm}^2$

$A_{sc} = \frac{m_2}{(1.5m-1)\sigma_{cbc} \left(\frac{x-d'}{x}\right)(d-d')}$
 $= \frac{26.5 \times 10^6}{(1.5 \times 13.33 - 1) \times 7 \left(\frac{116-40}{116}\right)(400-40)}$
 $= 844.7 \text{ mm}^2$

$A_{st2} = \frac{M_2}{\sigma_{st} (d-d')}$
 $= \frac{26.5 \times 10^6}{230 (360)}$
 $= 320$

Provide dia. of rod 16mm and 20mm.

$\eta_1 = \frac{A_{st1} \times y}{\pi \times d^2}$
 $= \frac{404.6 \times 4}{\pi \times 16^2}$
 $= 4.2 = 5$

$\eta_2 = \frac{A_{st2} \times y}{\pi \times d^2}$
 $= \frac{320 \times 4}{\pi \times 20^2}$
 $= 1.01$

$\eta = \frac{A_{sc}}{\pi/4 \times 20^2}$
 $= \frac{844.7}{\pi/4 \times 20^2} = 2.68 = 3$

$$A_{sc \text{ prov}} = n \times \frac{\pi}{4} \times \phi^2$$

$$= 3 \times \frac{\pi}{4} \times 20^2$$

$$= 942.47 \text{ mm}^2$$

$$A_{st \text{ prov}} = 3 \times \frac{\pi}{4} \times 16^2$$

$$= 603.18 \text{ mm}^2$$

$$A_{st \text{ prov}} = 3 \times \frac{\pi}{4} \times 20^2$$

$$= 942.47 \text{ mm}^2$$

$$A_{st} = 942.47 + 603.18$$

$$= 1545.65 \text{ mm}^2$$

Find the depth of actual neutral axis

$$\frac{bx^2}{2} + (1.5m - 1) A_{sc} (m - d') = m A_{st} (d - x)$$

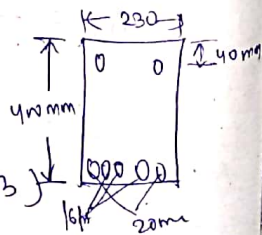
$$\Rightarrow \frac{230 x^2}{2} (1.5 \times 13.33 - 1) \times 942 (x - 40) - 13.33 \times 1545.65 (400 - x) = 0$$

$$\Rightarrow 115x^2 + 17893.3x - 715731.6 + 20603.5x - 8241405.8 = 0$$

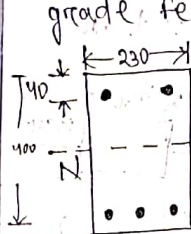
$$\Rightarrow x = 158 \text{ mm}$$

$$\Rightarrow x_{act} = 158 \text{ mm}$$

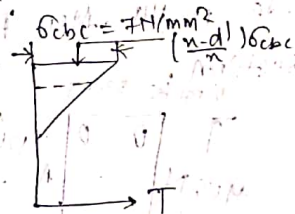
$$x_{act} < x_{uci} (1.2 \times 13)$$



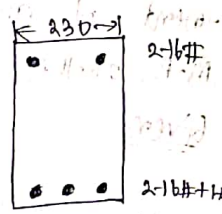
A rectangular beam 230mm wide x 400mm effective depth is subjected to a moment of 42 kNm. The effective cover of compressive reinforcement is 40mm. Find out the reinforcement steel. The material are m20 grade concrete and HYSD reinforcement of grade fe 415.



a) section



b) stress diagram



c) reinforcement

solⁿ

Given data

$$b = 230 \text{ mm}$$

$$d = 360 \text{ mm}$$

$$m = 42 \text{ kNm}$$

$$d' = 40 \text{ mm}$$

$$m_{20} = A_{st}$$

$$fe 415$$

$$m_1 = Q_{bal} \times b \times d^2$$

$$0.91 \times 230 \times 360^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

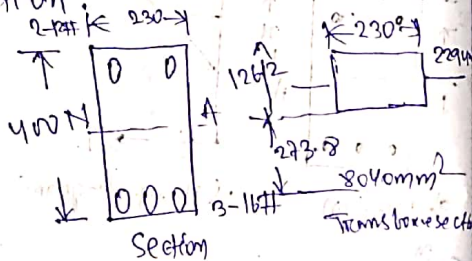
$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$A_{st} = ?$$

Example - 5.26

A rectangular beam is reinforced as shown in fig. Find out the maximum stress in concrete and steel if it is subjected to a moment of 42 kNm. The materials are M20 grade concrete and HYSD reinforcement of grade Fe 415. Also find out the M-R of the section.

Given



$$b = 230 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$d' = 25 \text{ mm}$$

$$M_{20} \text{ \& HYSD Fe 415}$$

$$M = 42 \text{ kNm}$$

$$m = \frac{280}{360} = 13.33$$

$$\sigma_{bc} = 9.4 \text{ N/mm}^2$$

$$\sigma_{st} = 230.4 \text{ N/mm}^2$$

$$k = \frac{m \sigma_{bc}}{\sigma_{st} + m \sigma_{bc}} = \frac{13.33 \times 9.4}{230.4 + 13.33 \times 9.4} = 0.29$$

$$\text{Compression steel } (A_{sc}) = n \times \frac{\pi}{4} \times d^2$$

$$= 2 \times \frac{\pi}{4} \times 12^2$$

$$= 226 \text{ mm}^2$$

$$\text{Transformed area} = (1.5m - 1) \times A_{sc}$$

$$= (1.5 \times 13.33 - 1) \times 226$$

$$= 4294 \text{ mm}^2$$

Tension steel

$$A_{st} = n \times \frac{\pi}{4} \times d^2$$

$$= 3 \times \frac{\pi}{4} \times 16^2$$

$$= 603 \text{ mm}^2$$

$$\text{Transformed area} = 13.33 \times 603 = 8040 \text{ mm}^2$$

To find the position of neutral axis taking moments of transformed area about neutral axis.

$$\frac{bx^2}{2} + (1.5m - 1) A_{sc} (n - d') = m A_{st} (d - x)$$

$$\Rightarrow \frac{230x^2}{2} + 4294(x - 25) = 8040(400 - x)$$

$$\Rightarrow 115x^2 + 4294x = 171760 - 8040x$$

$$\Rightarrow x = 126.2 \text{ mm}$$

$$x_{act} = 126.6 \text{ mm}$$

$$x_{crit} = kd$$

$$= 0.29 \times 400 = 116 \text{ mm}$$

$$x_{act} > x_{crit}$$

(The section is over-reinforced and concrete will fail 1st).

Taking moment of compressive forces about tensile steel and equating to the external B.M.

$$M = (1.5m-1) A_{sc} \left(\frac{x-d'}{x} \right) \sigma_{cbc} \times (d-d') + m \frac{\sigma_{sc}}{2} \left(d - \frac{x}{3} \right)$$

$$\Rightarrow 42 \times 10^6 = (1.5 \times 13.33 - 1) \times 226 \times \left(\frac{126.2 - 40}{126.2} \right) \times \sigma_{cbc} \times (400 - 40) + 230 \times 126.2 \times \left(400 - \frac{126.2}{3} \right) \times \frac{\sigma_{sc}}{2}$$

$$\Rightarrow 42 \times 10^6 = 1055597 \sigma_{cbc} + 5194687 \sigma_{sc}$$

$$\Rightarrow \sigma_{cbc} = 6.72 \text{ N/mm}^2$$

Stress in compression steel

$$\begin{aligned} \sigma_{sc} &= 1.5m \left(\frac{x-d'}{x} \right) \sigma_{cbc} \\ &= 1.5 \times 13.33 \times \left(\frac{126.2 - 40}{126.2} \right) \times 6.72 \\ &= 91.84 \text{ N/mm}^2 \end{aligned}$$

Stress in tension steel

$$\begin{aligned} \sigma_{st} &= m \sigma_{cbc} \left(\frac{d-d'}{x} \right) \\ &= 13.33 \times 6.72 \times \left(\frac{400 - 126.2}{126.2} \right) \end{aligned}$$

$$= 194.35 \text{ N/mm}^2$$

M.R for compression

$$= \frac{.7}{6.72} \times 42 = 43.75 \text{ kNm}$$

$$\text{M.R for tension} = \frac{280}{194.35} \times 42 = 49.7 \text{ kNm}$$

$$M.R = 43.75 \text{ kNm}$$

Example - 5.38

A rectangular beam is reinforced as shown in figure. Find out the moment of resistance of the section. The materials are M20 grade concrete and mild steel reinforcement.

Given data

$$b = 230 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$d' = 40 \text{ mm}$$

M20 & mild steel reinforcement

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 140 \text{ N/mm}^2$$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402 \text{ mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603 \text{ mm}^2$$

$$\text{Transformed area of compression steel} \\ = (1.5m-1) \cdot A_{sc}$$

$$= (1.5 \times 12.33 - 1) \times 402 = 7636 \text{ mm}^2$$

$$\text{Transformed area of tension steel}$$

$$= m A_{st}$$

$$= 12.33 \times 603$$

$$= 8038 \text{ mm}^2$$

To find the neutral axis, taking moment

$$\frac{bx^2}{2} + (1.5m-1) A_{sc} (x-d') = m A_{st} (d-x)$$

$$\Rightarrow \frac{230x^2}{2} + 7636(x-40) = 8038(450-x)$$

$$\Rightarrow 115x^2 + 7636x - 305440 = 3619100 - 8038x = 0$$

$$\Rightarrow x = 128.7 \text{ mm}$$

$$\Rightarrow x_{act} = 128.7 \text{ mm}$$

Depth of balanced H.A.

$$x_{bal} = k_d d$$

$$= 0.90 \times 450$$

$$= 180 \text{ mm}$$

$$x_{act} < x_{bal}$$

The beam is under-reinforced and steel will fail first. Then find the lever arm, eq. of the compressive forces is to be found out.

$$C_1 = (1.5m-1) A_{sc} \times \left(\frac{x-d'}{x} \right) f_{cb}$$

$$= 7636 \times \frac{128.7-40}{128.7} \times f_{cb}$$

$$= 5262 f_{cb}$$

$$C_2 = bx \times \frac{f_{cb}}{2}$$

$$= 230 \times 128.7 \times \frac{f_{cb}}{2}$$

$$= 14800 f_{cb}$$

$$y_1 = 40 \text{ mm}, y_2 = \frac{128.7}{2} = 42.9 \text{ mm}$$

$$\bar{y} = \frac{5262 f_{cb} \times 40 + 14800 f_{cb} \times 42.9}{5262 f_{cb} + 14800 f_{cb}}$$

$$= 42.14 \text{ mm}$$

$$\text{Lever arm} = d - \bar{y}$$

$$= 450 - 42.14$$

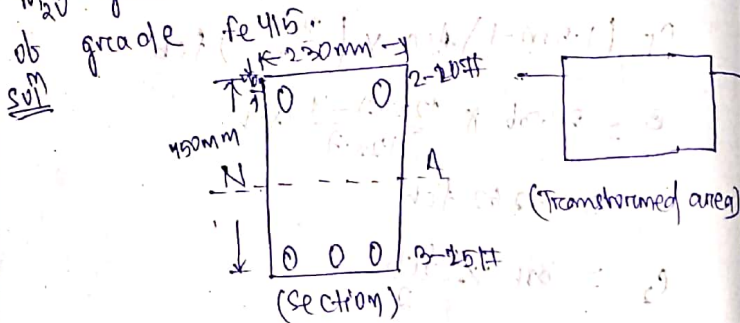
$$= 407.86 \text{ mm}$$

$$M_R = A_{st} \cdot \sigma_{st} \cdot j_d$$

$$= 140 \times 603 \times 407.86 \times 10^{-6} = 34.43 \text{ kNm}$$

Example -5.37

A rectangular beam is reinforced as shown in figure. Find out the moment of resistance of the section. The materials are M20 grade concrete and HYSD reinforced steel.



$$b = 230 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$d' = 40 \text{ mm}$$

M20 & HYSD Fe415

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$m = \frac{230}{7} = 13.33$$

$$k = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}} = 0.29$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2 = 628 \text{ mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 25^2 = 1472 \text{ mm}^2$$

Transformed area of compression steel

$$= (1.5 - 1) A_{sc}$$

$$= (1.5 \times 13.33 - 1) \times 628 = 11930 \text{ mm}^2$$

Transformed area of tension steel

$$= m A_{st}$$

$$= 13.33 \times 1472$$

$$= 19630 \text{ mm}^2$$

To find the depth of actual neutral axis

$$\frac{bx^2}{2} + (1.5m - 1) A_{sc} (x - d') = m A_{st} (d - x)$$

$$\Rightarrow \frac{230x^2}{2} + 11930 \times (x - 40) = 19630 (450 - x)$$

$$\Rightarrow 115x^2 + 11930x = 477250 + 19630x - 8833500 = 0$$

$$\Rightarrow x = 178.7 \text{ mm}$$

Depth of balanced neutral axis

$$= kd = 0.29 \times 450$$

$$= 130.5 \text{ mm}$$

Max x > x_{bal}

The section is over reinforced and concrete stress reaches the maximum value first.

$$M_R = M_1 + M_2$$

$$M_1 = C_1 \times \text{lever arm}$$

$$= \frac{1}{2} \sigma_{cbc} b \cdot x \left(d - \frac{x}{3} \right)$$

$$= \frac{1}{2} \times 7 \times 230 \times 178.7 \left(450 - \frac{178.7}{3} \right) \times 10^{-6}$$

$$= 56.16 \text{ kNm}$$

$$M_2 = C_2 \times \text{lever arm}_2$$

$$= (1.5m - 1) A_{sc} \cdot \sigma_{cbc} \left(\frac{x - d'}{x} \right) (d - d')$$

$$= (1.5 \times 13.33 - 1) \times 628 \times 7 \left(\frac{178.7 - 40}{178.7} \right) (450 - 40) \times 10^{-6}$$

$$= 26.58 \text{ kNm}$$

$$M = M_1 + M_2$$

$$= 56.16 + 26.58$$

$$= 82.74 \text{ kNm}$$

Example - 25.29

A rectangular beam is reinforced as shown in figure find out the moment of resistance of the section. The material are M_{20} grade concrete and HYSD reinforcement of grade 415.

Given data

$$b = 230 \text{ mm}$$

$$d = 560 \text{ mm}$$

$$d' = 40 \text{ mm}$$

$$M_{20} \text{ \& HYSD Fe 415}$$

$$\sigma_{cbc} = 9.41 / \text{mm}^2$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

Transformed area of compression of steel

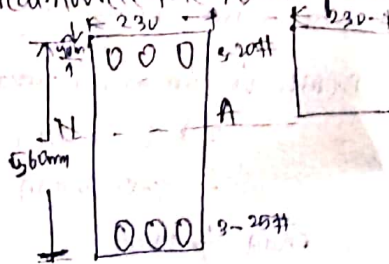
$$(1.5m-1) A_{sc} (1.5 \times 13.33 - 1) 941$$

$$= 12894 \text{ mm}^2$$

Transformed area of tension steel

$$= m A_{st} = 13.33 \times 1473$$

$$= 19635 \text{ mm}^2$$



$$m = \frac{280}{36 \text{ chr}} = 13.33$$

$$A_{sc} = 3 \times \frac{\pi}{4} 20^2 = 941 \text{ mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} 20^2 = 941 \text{ mm}^2$$

To find the depth of neutral axis

$$\frac{b x^2}{2} + (1.5m-1) A_{sc} (x-d') = m A_{st} (d-x)$$

$$\Rightarrow \frac{230 x^2}{2} + 12894 x - 12894 \times 40 = 19635 (560 - x)$$

$$\Rightarrow 115 x^2 + 12894 x - 515360 + 19635 x - 10999800 = 0$$

$$\Rightarrow x = 195.2 \text{ mm}$$

Depth of critical neutral axis

$$= x_c = 0.29 \times 560$$

$$= 162.4 \text{ mm}$$

$$x_{act} > x_{crit}$$

The beam is over-reinforced & concrete will fail first.

$$M_R = m_1 + m_2$$

$$m_1 = \frac{\sigma_{cbc}}{2} (b \times x (d - \frac{x}{3}))$$

$$= \frac{9.41}{2} \times 230 \times 195.2 (560 - \frac{195.2}{3})$$

$$= 77.78 \text{ kNm}$$

$$m_2 = (1.5m-1) A_{sc} \cdot \sigma_{cbc} (\frac{x-d'}{x}) (d-d')$$

$$= 12894 \times 9.41 (\frac{195.2-40}{195.2}) \times (560-40) 10^6$$

$$= 51.78 \text{ kNm}$$

$$M_R = m_1 + m_2$$

$$= 77.78 + 51.78$$

$$= 129.56 \text{ kNm}$$

LIMIT STATE METHOD DL-14.08.18

Grade of concrete

, , , steel

maintain at time, cur, b.m with suitable w/c ratio
M₂₀, M₂₅ etc
F₂₅₀, f_e 415 etc
characteristic strength of concrete
strength of conc & steel

$$f_{ck} = 20 \text{ N/mm}^2 = \frac{20}{3} = 6.67 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2 = \frac{415}{1.2} = 228.8 = 230 \text{ N/mm}^2$$

In limit state method more attention is given to the acceptable limits before ultimate load theory.

Limit load:-

The maximum load that can be applied to a structure before failure or collapse begins.

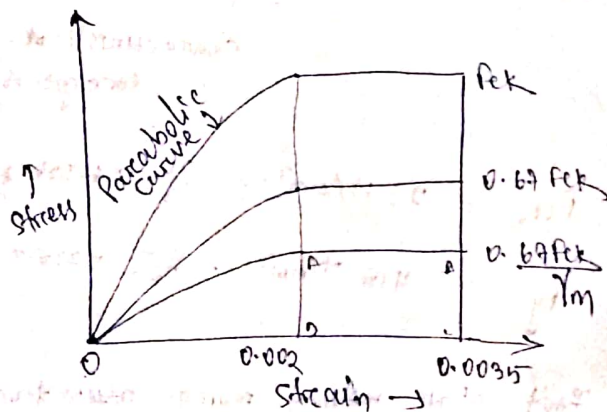
Assumption of limit state method:-

i) plane section normal to the axis remain plane after bending.

ii) The maximum strain in concrete at the outer most compression fibre is

taken as 0.0025 in bending.

iii) The stress strain curve of concrete is parabolic and the strain value 0.002

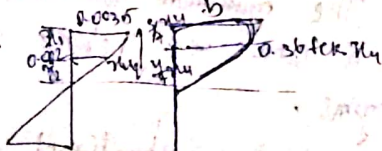


$\gamma_m = \text{Factor of Safety} = 1.5$

$$\frac{0.67 f_{ck}}{1.5} = 44.6 \text{ MPa}$$

Note $\sigma_0 = \sigma_2 = \frac{0.002 \sigma_u}{0.0035} = \frac{2}{2.5} \sigma_u = \frac{4}{5} \sigma_u$

$\sigma_1 = \sigma_u - \frac{4}{5} \sigma_u = \frac{1}{5} \sigma_u$
 $= \frac{3}{5} \sigma_u$



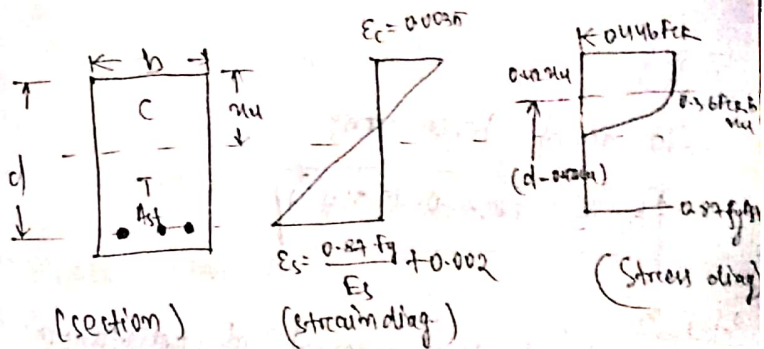
Area of Paraboloid = $\frac{2}{3} \times \frac{4}{5} \sigma_u \times 44.6 \text{ MPa}$
 $= 0.17 \text{ fck} \sigma_u$

Area of Rect. = $0.446 \text{ fck} \times \frac{3}{5} \sigma_u$
 $= 0.19 \text{ fck} \sigma_u$

Total area = $0.36 \text{ fck} \sigma_u$

$$\bar{\sigma} = \frac{(0.17 \text{ fck} \sigma_u) (\sigma_1 + \frac{3}{5} \sigma_2) + 0.19 \text{ fck} \sigma_u (\frac{3}{5} \sigma_2)}{0.36 \text{ fck} \sigma_u}$$

SINGLY REINFORCED



f_y	$\sigma_u \text{ MPa}$
250	0.53 d
415	0.48 d
510	0.46 d
550	0.44 d

Step-1

To find depth of neutral axis

$$0.36 f_{ck} b x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

Step-2

To find lever arm

$$z = (d - 0.42 x_u)$$

Step-3

To find the moment of resistance

$$M = \text{compressive force} \times \text{lever arm} \\ = \text{tension force} \times \text{lever arm}$$

from both under reinforced & over reinforced

$$M = 0.36 f_{ck} b x_u (d - 0.42 x_u) \\ = 0.36 f_{ck} b d \left(1 - \frac{0.42 x_u}{d}\right) \frac{x_u}{d} \\ = 0.36 \frac{x_u}{d} (1 - 0.42 \frac{x_u}{d}) f_{ck} b d^2$$

for limiting section

$$M_{u \text{ limit}} = 0.36 f_{ck} \frac{x_{u \text{ max}}}{d} \left(1 - \frac{0.42 x_{u \text{ max}}}{d}\right) b d^2$$

$$m_{u \text{ lim}} = Q_{\text{lim}} b d^2$$

$$Q_{\text{lim}} = \frac{M_{u \text{ lim}}}{b d^2}$$

104-12.02.18

Q. Find out the value of moment resistance factor for M20 grade concrete and Fe415 HYSD reinforcement.

solⁿ

Given data

$$\frac{x_{u \text{ max}}}{d} = 0.48$$

$$M_{20} = 20 \text{ N/mm}^2$$

$$Fe_{415} = f_y = 415 \text{ N/mm}^2, \quad \frac{x_{u \text{ max}}}{d} = 0.48$$

$$M_{u \text{ lim}} = 0.36 f_{ck} \frac{x_{u \text{ max}}}{d} \left(1 - \frac{0.42 x_{u \text{ max}}}{d}\right) b d^2$$

$$Q_{\text{lim}} = 0.36 f_{ck} \frac{x_{u \text{ max}}}{d} \left(1 - \frac{0.42 x_{u \text{ max}}}{d}\right) \\ = 0.36 \times 20 \times 0.48 (1 - 0.42 \times 0.48) \\ = 2.759 = 2.76$$

for mild steel
 $m_{20} = 20 \text{ N/mm}^2$

$$f_{e20} = f_y = 250 \text{ N/mm}^2, \quad x_u \text{ max} = 0.53$$

$$m_u \text{ term} = 0.36 f_{ck} \frac{x_u \text{ max}}{d} \left(1 - 0.42 \frac{x_u \text{ max}}{d}\right) b d$$

$$Q_{1 \text{ term}} = 0.36 f_{ck} \frac{x_u \text{ max}}{d} \left(1 - 0.42 \frac{x_u \text{ max}}{d}\right)$$

$$= 0.36 \times 20 \times 0.53 \left(1 - 0.42 \times 0.53\right)$$

$$\boxed{Q_{\text{sum}} = 2.96}$$

* Under reinforced section:-

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$\boxed{M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b d}\right)}$$

Divides both sides by $0.87 f_y b d^2$

$$\frac{m_u}{0.87 f_y b d^2} = \frac{A_{st}}{b d} \left(1 - \frac{A_{st} f_y}{f_{ck} b d}\right)$$

$$\left[\frac{f_y}{m_u} = \frac{A_{st}}{b d} \right]$$

$$\text{Here } A_{st} = \frac{P_t b d}{100}$$

$$\frac{P_t}{100} = \frac{A_{st}}{b d}$$

$$\frac{m_u}{0.87 f_y b d^2} = \frac{P_t}{100} \left(1 - \frac{P_t f_y}{f_{ck}} \left(\frac{P_t}{100}\right)\right)$$

$$= \frac{P_t}{100} - \frac{f_y}{f_{ck}} \left(\frac{P_t}{100}\right)^2$$

$$\frac{P_t}{100} = \frac{1 \pm \sqrt{1 - \frac{4.6 m_u}{f_{ck} b d^2}}}{2 \times \frac{f_y}{f_{ck}}}$$

$$\boxed{P_t = 50 \left[1 - \sqrt{1 - \frac{4.6 m_u}{f_{ck} b d^2}} \right] \frac{f_y}{f_{ck}}}$$

Q. A rectangular beam of width 230mm and effective depth of 520mm is reinforced with 4-16^{mm} ^{tensile} reinforcement. Find out the depth of neutral axis specify the type of beam and find out the moment of the section. The materials are M20 grade concrete & Fe415 HYSD reinforcement.

⑩ 4-20#

Solⁿ Given data

$$b = 230 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$A_{st} = 4-16\# = 804 \text{ mm}^2 \quad \mu_{u \times \frac{1}{4} \times b^2}$$

$$f_{e415} = f_y = 415 \text{ N/mm}^2$$

$$m_{20} = 20 \text{ N/mm}^2 \text{ (fck)}$$

To find the depth of neutral axis

$$\mu_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$\mu_u = \frac{0.87 \times 415 \times 804}{0.36 \times 20 \times 230}$$

$$= 175.29 \text{ mm}$$

find lever arm

$$z = (d - 0.42 \mu_u)$$

$$= (520 - 0.42 \times 175.29)$$

$$= 446.37 \text{ mm}$$

Step 2

To find the moment of resistance

$$M = \text{compressive force} \times \text{lever arm}$$

$$M_u = 0.36 \times 20 \times 230 \times 175.29 (446.37)$$

$$= 129.57 \text{ kNm}$$

$$P_t = 50 \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}} \right]$$

$$f_y / f_{ck}$$

$$= 0.67 \times 100 = 67\%$$

Solⁿ-2

$$b = 230 \text{ mm } d = 520 \text{ mm } \mu_{u \text{ max}} = 0.48$$

$$A_{st} = 4-20\# = 1256 \text{ mm}^2 = 0.48 \times 520$$

$$f_{e415} = f_y = 415 \text{ N/mm}^2 = 249.6$$

$$m_{20} = 20 \text{ N/mm}^2 \text{ (fck)}$$

To find the depth of neutral axis

$$\mu_u = \frac{0.87 \times 415 \times 1256}{0.36 \times 20 \times 230} = 273.83$$

$$\mu_{u \text{ max}} = 0.48 \times d = 0.48 \times 520 = 249.6 \text{ mm}$$

1. Find the lever arm $x_u \rightarrow x_{u, \max}$ (over reinforcement)

$$z = (d - 0.42 x_u)$$

$$= (520 - 0.42 \times \frac{249.6}{25})$$

$$= \cancel{404.99} \text{ mm} \quad 415.16 \text{ mm}$$

To find the moment of resistance

$m_u = \text{compressive force} \times \text{lever arm}$

$$M_u = 0.36 f_{ck} b x_{u, \max} (d - 0.42 x_{u, \max})$$

$$= 0.36 \times 20 \times 230 \times \frac{249.6}{25} \times \frac{415.16}{25}$$

$$= \cancel{123.69} \text{ kNm} \quad 171.6 \text{ kNm}$$

$$M_u = 171.6 \text{ kNm}$$

$$R_t = 50 \left[1 - \sqrt{1 - \frac{4.6 \times m_u}{f_{ck} b d^2}} \right]$$

$$\frac{f_y}{f_{ck}}$$

$$= 50 \left[1 - \sqrt{1 - \frac{4.6 \times 171.6 \times 10^6}{20 \times 230 \times 520^2}} \right]$$

$$\frac{415}{20} = 0.954 = 95.4\%$$

Assignment -

A singly reinforced rectangular beam of width 230 mm and effective depth 460 mm is reinforced with 3-20 mm dia bars find out the moment of resistance and percentage of steel. And also find out the max grade concrete & Fe 500 HYSD reinforcement

solⁿ Given data:- 3-20 mm dia bars

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$A_{st} = 3 \times 20 \text{ mm dia bars} = 3 \times \frac{\pi}{4} \times 20^2$$

$$= 942 \text{ mm}^2$$

$$f_{e 500} = f_y = 500 \text{ N/mm}^2$$

$$m_{25} = 25 \text{ N/mm}^2 \text{ (fck)}$$

To find the depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 500 \times 942}{0.36 \times 25 \times 230}$$

$$= 199.95 \text{ mm}$$

$$\begin{aligned} \eta_{u \max} &= 0.46 d \\ &= 0.46 \times 460 \\ &= 211.6 \text{ mm} \end{aligned}$$

$\eta_u < \eta_{u \max}$ (The section is underreinforced)

$$\begin{aligned} M_u &= 0.87 f_y A_{st} (d - 0.42 \eta_u) \\ &= 0.87 \times 500 \times 942 (460 - 0.42 \times 197.95) \\ &= 154426332 \text{ Nmm} \\ &= 154.42 \text{ kNm} \end{aligned}$$

$$\begin{aligned} P_t &= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right] \\ &= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 154.42 \times 10^6}{25 \times 230 \times 460^2}}}{250 / 25} \right] \\ &= 0.887 = 88.7\% \end{aligned}$$

For 5-20 mm dia bars:-

$$\begin{aligned} b &= 230 \text{ mm} \\ d &= 460 \text{ mm} \end{aligned}$$

$$\begin{aligned} A_{st} &= 5-20 \text{ dia bars} = 5 \times \frac{\pi}{4} \times 20^2 \\ &= 1570 \text{ mm}^2 \end{aligned}$$

$$f_{e500} = f_y = 500 \text{ N/mm}^2$$

$$M_{25} = 25 \text{ N/mm}^2 \text{ (fck)}$$

To find the depth of neutral axis

$$\begin{aligned} \eta_u &= \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \\ &= \frac{0.87 \times 500 \times 1570}{0.36 \times 25 \times 230} \\ &= 329.92 \text{ mm} \end{aligned}$$

$$\begin{aligned} \eta_{u \max} &= 0.46 d \\ &= 0.46 \times 460 \\ &= 211.6 \text{ mm} \end{aligned}$$

$\eta_u > \eta_{u \max}$ (The section is overreinforced)

$$\begin{aligned} M_u &= 0.87 f_y A_{st} (d - 0.42 \eta_{u \max}) \\ &= 0.87 \times 500 \times 1570 (460 - 0.42 \times 211.6) \\ &= 251847460 = 251.85 \text{ kNm} \end{aligned}$$

$$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 251.84 \times 10^6}{25 \times 230 \times 460^2}}}{2500 / 25} \right]$$

$$= 0.953 = 95.3\%$$

Q. A singly reinforced rectangular beam of width 250mm and 520mm effective depth is reinforced with 4-16mm dia bars find the moment of resistance and required percentage of steel. M_{u0} Fe 550

Given data:-

$$b = 250 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$A_{st} = 4-16 \text{ dia bars}$$

$$= 4 \times \frac{\pi}{4} \times 16^2 = 804 \text{ mm}^2$$

$$f_{ck} = 30 \text{ N/mm}^2$$

$$f_{e550} = f_y = 550 \text{ N/mm}^2$$

Find the depth of neutral axis

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 550 \times 804}{0.36 \times 30 \times 250}$$

$$= 142.48 \text{ mm}$$

$$x_{u \text{ max}} = 0.44 d$$

$$= 0.44 \times 520$$

$$= 228.8 \text{ mm}$$

$x_u < x_{u \text{ max}}$ (The section is under reinforced)

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 550 \times 804 (520 - 0.42 \times 142.48)$$

$$= 177.02 \text{ kNm}$$

$$P_t = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 177.02 \times 10^6}{30 \times 250 \times 520^2}}}{550 / 30} \right]$$

$$0.6174 \times 100 = 61.74\%$$

Designed Problem

A singly reinforced rectangular beam subjected to a bending moment of 36 kNm working load the width of the beam is 200 mm wide. Find the depth of the beam and the steel area. The materials are M20 grade concrete and mild steel reinforcement.

Sol Given data

$$b = 200 \text{ mm}$$

$$d = ?$$

$$A_{st} = ?$$

$$W_L = 36 \text{ kNm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$M_u = 1.5 \times 36 \text{ kNm} = 54 \text{ kNm}$$

$$Q_{lim} = 0.36 f_{ck} \frac{x_{u, \max}}{d} \left(1 - 0.42 \frac{x_{u, \max}}{d} \right)$$

$$= 0.36 \times 20 \times 0.53 (1 - 0.42 \times 0.53)$$

$$= 2.96$$

$$\frac{x_{u, \max}}{d} = 0.53$$

$$d^2 = \frac{M_u}{Q_{lim} b}$$

$$d = \sqrt{\frac{M_u}{Q_{lim} b}}$$

$$= \sqrt{\frac{54 \times 10^6}{2.96 \times 200}}$$

$$= 302.22 \text{ mm}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - 0.42 \frac{x_{u, \max}}{d} \right)$$

$$A_{st} = \frac{M_u}{0.87 f_y d \left(1 - 0.42 \frac{x_{u, \max}}{d} \right)}$$

$$= \frac{54 \times 10^6}{0.87 \times 250 \times 302 (1 - 0.42 \times 0.53)}$$

$$= 1057 \text{ mm}^2$$

$$P_{t\lim} = 50 \left[1 - \frac{\sqrt{1 - 4.6 \frac{P_{t\lim}}{f_{ck}}}}{f_y / f_{ck}} \right]$$

$$A_{st\lim} = \frac{P_{t\lim} b d}{f_y}$$

$$P_{t\lim} = 50 \left[1 - \frac{\sqrt{1 - 4.6 \times \frac{20}{250 \times 20}}}{250 / 20} \right]$$

$$P_{t\lim} = 1.75$$

$$A_{st\lim} = \frac{P_{t\lim} b d}{f_y} = \frac{1.75 \times 302 \times 302}{250} = 1057 \text{ mm}^2$$

$$A_{st\text{prov}} = 1143 \text{ mm}^2$$

$$d = 302 \text{ mm}$$

$$D = d + \frac{\phi}{2} + c.e = 302 + \frac{20}{2} + 30 = 342 \text{ mm}$$

DT-25-08-18

Q. Design a simply reinforced rectangular beam for a factored moment of 120 kNm. Assume the width of the beam as 230 mm. The materials are M20 grade concrete & Fe415 HYSD reinforcement.

Soln Given data

$$b = 230 \text{ mm}$$

$$d = ? = 435 \text{ mm}$$

$$A_{st} = ?$$

$$M_u = 120 \text{ kNm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$D_{\text{reqd}} = 435 + 10 + 30 = 475 \text{ mm}$$

$$D_{\text{prov}} = 500 \text{ mm}$$

$$d_{\text{prov}} = 500 - 10 - 30 = 460 \text{ mm}$$

then $P_{t\text{reqd}} =$

$$M_u = 120 \text{ kNm}$$

$$Q_{\lim} = 0.36 f_{ck} \frac{x_{u\max}}{d} \left[1 - \frac{0.42 x_{u\max}}{d} \right]$$

$$= 0.36 \times 20 \times 0.48 \left(1 - 0.42 \times 0.48 \right)$$

$$= 2.75$$

$$d^2 = \frac{M_u}{Q_{\lim} b} \Rightarrow d = \sqrt{\frac{M_u}{Q_{\lim} b}}$$

$$= \sqrt{\frac{120 \times 10^6}{2.75 \times 230}} = 435 \text{ mm}$$

$$\downarrow P_{t \text{ lim}} = 50 \left[\frac{1 - \sqrt{1 - 4.6 \times \frac{Q_{\text{lim}}}{f_{ck}}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - 4.6 \times \frac{2.75}{20}}}{415 / 20} \right]$$

$$A_{st} = \frac{M_u}{0.87 f_y d \left(1 - 0.42 \frac{\mu_{\text{max}}}{d} \right)}$$

$$= \frac{1.20 \times 10^6}{0.87 \times 415 \times 435 (1 - 0.42 \times 0.48)}$$

$$= 957 \text{ mm}^2 \quad A_t = 50 \left[\frac{1 - 4.6 \times \frac{M_u}{f_{ck} b d^2}}{f_y / f_{ck}} \right]$$

$$P_{t \text{ req}} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 1.20 \times 10^6}{20 \times 230 \times 435^2}}}{415 / 20} \right]$$

$$0.824 = 0.824$$

$$A_{st \text{ req}} = P_{t \text{ req}} = \frac{P_{t \text{ req}} b d}{100}$$

$$= \frac{0.824 \times 230 \times 435}{100}$$

$$= 950 \text{ mm}^2$$

$$A_{st \text{ lim}} =$$

$$A_{st \text{ req}} < A_{st \text{ prov}} < A_{st \text{ lim}}$$

$$A_{st \text{ req}} = \frac{P_{t \text{ req}} b d}{100}$$

$$= \frac{0.824 \times 230 \times 460}{100}$$

$$= 872 \text{ mm}^2$$

$$P_{t \text{ lim}} = 0.95$$

$$A_{st \text{ lim}} = \frac{0.95 \times 230 \times 460}{100}$$

$$= 1005 \text{ mm}^2$$

$$A_{st \text{ prov}} = \frac{A_{st \text{ req}}}{\pi/4 \times d^2}$$

$$= \frac{872}{\pi/4 \times 314} \approx 3 \text{ nos}$$

$$A_{st \text{ prov}} = 942 \text{ mm}^2$$

Q. Design a singly reinforced rectangular cantilever beam of 230mm width and 550mm effective depth, is subjected to a bending moment of 80kNm at working load. Find the steel area required the material are m20 grade concrete & Fe415 HYSD reinforcement.

Given data

$$b = 230 \text{ mm}$$

$$d = 550 \text{ mm}$$

$$M_L = 80 \text{ kNm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$M_u = 1.5 \times 80 = 120 \text{ kNm}$$

$$Q_{lim} = 0.36 f_{ck} \frac{x_u \text{ max}}{d} \left(1 - \frac{0.42 x_u \text{ max}}{d} \right)$$

$$= 0.36 \times 20 \times 0.48 \left(1 - 0.42 \times 0.48 \right)$$

$$= 2.76$$

$$D_{req} = 550 + 10 + 50$$

$$= 610 \text{ mm}$$

$$P_{t \text{ req}} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 120 \times 10^6}{20 \times 230 \times 550^2}}}{415 / 20} \right]$$

$$= 0.672$$

$$P_{t \text{ lim}} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times Q_{lim}}{f_{ck}}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 2.76}{20}}}{415 / 20} \right]$$

$$= 0.95$$

$$A_{st_{um}} = \frac{0.95 \times 230 \times 103570}{100}$$

$$= 1092 \text{ mm}^2$$

~~Ast prov (mm)~~

$$A_{st_{req}} = \frac{P_{t_{req}} b d}{100}$$

$$= \frac{0.692 \times 230 \times 500}{100}$$

$$= 771 \text{ mm}^2$$

$$A_{st_{prov}} (mm) = \frac{A_{st_{req}}}{\pi/4 d^2}$$

$$= \frac{771}{\pi/4 (10)^2} = \frac{771}{314}$$

$$= \frac{771}{\pi/4 \times 10} = 2.45 \approx 3$$

$$= 942 \text{ mm}^2$$

$$b = 250 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_{yk} = 500 \text{ N/mm}^2$$

$$M_u = 90 \text{ kNm}$$

$$M_u = 1.5 \times 90 = 135 \text{ kNm}$$

$$Q_{um} = 0.36 f_{ck} \frac{x_{u_{max}}}{d} \left(1 - 0.42 \frac{x_{u_{max}}}{d} \right)$$

$$= 0.36 \times 25 \times 0.46 (1 - 0.42 \times 0.46)$$

$$= 3.34$$

$$P_{t_{req}} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_{yk} / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 135 \times 10^6}{25 \times 250 \times 520^2}}}{500 / 25} \right]$$

$$= 0.511$$

$$P_{lim} = 50 \left[1 - \sqrt{1 - \frac{4.6 Q_{lim}}{f_{ck}}} \right] \frac{f_y}{f_{ck}}$$

$$= 50 \left[1 - \sqrt{1 - \frac{4.6 \times 3.34}{25}} \right] \frac{500}{25}$$

$$= 0.94$$

$$A_{st req} = \frac{P_{req} b d}{100}$$

$$= \frac{0.511 \times 250 \times 520}{100}$$

$$= 864 \text{ mm}^2$$

$$A_{st lim} = \frac{0.94 \times 250 \times 520}{100}$$

$$= 1222$$

$$A_{st prov} (m) = \frac{A_{st req}}{1/4 d^2}$$

$$A_{st req} < A_{st prov} < A_{st lim}$$

$$664 < \quad < 1222$$

Q. Design a singly reinforced rectangular beam for an applied bending moment of 120 kNm at working loads. Assume the width of the beam as 250 mm. The materials are M25 grade concrete and Fe 500.

Given data

$$b = 250 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 500 \text{ N/mm}^2$$

$$M_u = 1.5 \times 120 \text{ kNm} = 180 \text{ kNm}$$

$$Q_{lim} = 0.36 f_{ck} \frac{x_{u max}}{d} \left(1 - \frac{0.42 x_{u max}}{d} \right)$$

$$= 0.36 \times 25 \times 0.46 \left(1 - 0.42 \times 0.46 \right)$$

$$= 3.34$$

$$d^2 = \frac{M_u}{\rho_{st} b} \Rightarrow d = \sqrt{\frac{M_u}{Q_{lim} b}}$$

$$= \sqrt{\frac{180 \times 10^6}{3.34 \times 250}} = 464 \text{ mm}$$

$$D_{req} = 469 + 10 + 20$$

$$P_{treq} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 180 \times 10^6}{25 \times 250 \times 469^2}}}{570 / 25} \right]$$

$$= 0.87$$

$$P_{t,um} = 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 Q_{um}}{f_{ck}}}}{f_y / f_{ck}} \right]$$

$$= 50 \left[\frac{1 - \sqrt{1 - \frac{4.6 \times 3.34}{25}}}{570 / 25} \right]$$

$$= 0.94$$

$$A_{st, req} = \frac{P_{treq} b d}{100}$$

$$= \frac{0.95 \times 250 \times 469}{100} = 1090 \text{ mm}^2$$

$$A_{st, um} = \frac{0.94 \times 250 \times 469}{100}$$

$$A_{st} = 1090 \text{ mm}^2$$

$$A_{st, prov} (m) = \frac{A_{st, req}}{\pi/4 (d^2)}$$

Doubly Reinforced of Beam

10-28-08-18

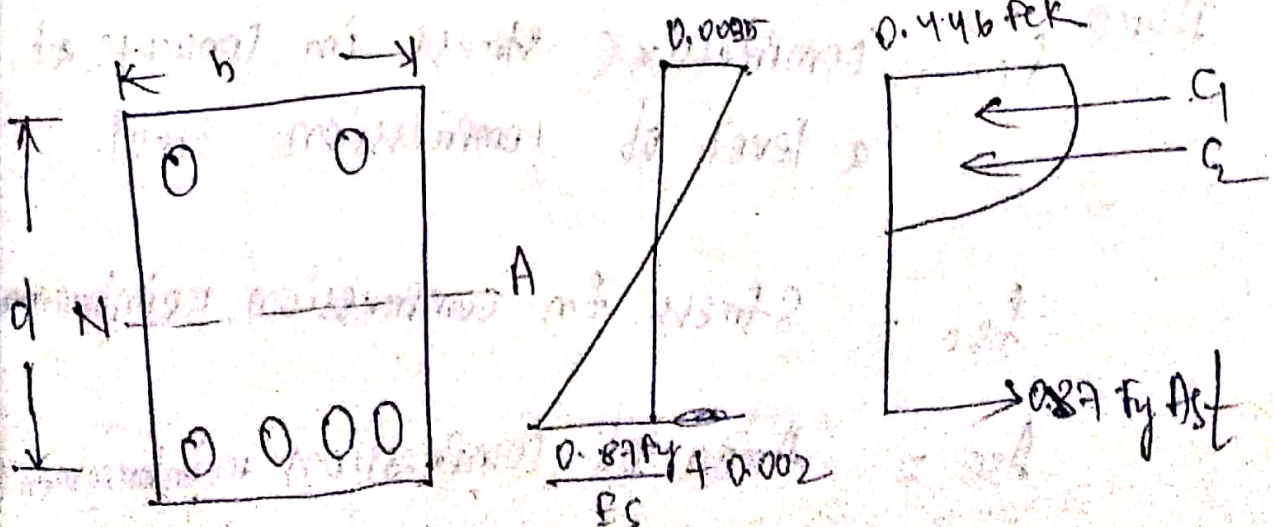
If the applied moment is greater than the moment of resistance of singly reinforced beam then there can be three possible way to increase the strength of the section.

1. Increase the dimension of the section especially depth of the section.

2. Higher grade of concrete can be used to increase the M.R of the section.

3. Introduce of steel reinforcement on the compression zone of the beam. and that is known as doubly reinforced beam.

$$M_u = M_{u1} + M_{u2}$$



Step-1

To find the depth of neutral axis

$$C_1 = 0.36 f_{ck} b x_u$$

$$C_2 = A_{sc} (f_{sc} - f_{cc})$$

$$T = 0.87 f_y A_{st}$$

~~$$0.36 f_{ck} b x_u + 0.87 f_y A_{st}$$~~

$$0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - A_{sc} (f_{sc} - f_{cc})}{0.36 f_{ck} b}$$

$$f_{cc} = 0.446 f_{ck}$$

$$f_{sc} = \frac{d'}{d} \text{ ratio up to } 0.2$$

Here f_{cc} = compressive stress in concrete at a level of compression steel.

f_{sc} = stress in compression reinforcement

A_{sc} = Area of compression reinforcement

Step-2 M_{u1}

To find the moment of resistance of the section

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} (f_{sc} - f_{cc}) (d - d')$$

Step-3

To find out reinforcement bar a given section and factored moment.

i) find out the m_{um} & A_{ctum} .

ii) obtained (find) the value of $m_u = m_u - m_{um}$

iii) find out compression steel

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

$$A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')}$$

$$A_{st2} = \frac{A_{sc} \cdot f_{sc}}{0.87 f_y}$$

Q. Find the factored moment of resistance (M_{or}) of a beam section having width 230 mm and 460 mm effective depth. The beam is reinforced with 2-16 mm dia compressive reinforcement having an effective cover of 40 mm & 4-20 mm dia tension reinforcement. The materials are M20 concrete & Fe25.

Solⁿ

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$d' = 40 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 250 \text{ N/mm}^2$$

$$A_{sc} = 2-16\phi = 402 \text{ mm}^2, A_{st} = 4-20\phi$$

$$M_u = ?$$

$$M_u = M_{u1} + M_{u2}$$

Step-1

To find the depth of neutral axis

$$0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - A_{sc} (f_{sc} - f_{cc})}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 250 \times 1256 - 402 (208)}{0.36 \times 20 \times 230}$$

$$x_u = 114.45 \text{ mm}$$

$$x_{u \max} = 0.53 d$$

$$= 0.53 \times 460$$

$$= 243.8 \text{ mm}$$

$$x_u < x_{u \max} \text{ (VRS)}$$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} (f_{sc} - f_{cc}) \times (d - d')$$

$$= 0.36 \times 20 \times 230 \times 114.5 (460 - 0.42 \times 114.5)$$

$$+ 402 (208) \times (460 - 40)$$

$$= 113205180.1 \text{ Nmm}$$

$$= 113.20 \text{ kNm}$$

Q. Find the factored moment of resistance of a beam section 230 mm wide and 400 mm effective depth reinforced with 2-16 mm diameter bars as compression reinforcement at an effective cover of 40 mm and 4-20 mm diameter bars as tension reinforcement. The materials are M20 grade concrete and mild steel reinforcement.

Solⁿ
Given data

$$b = 230 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$d' = 40 \text{ mm}$$

$$A_{sc} = 2-16$$

$$= 2 \times 201 = 402 \text{ mm}^2$$

$$A_{st} = 4-20 = 1256 \text{ mm}^2$$

$$= 4 \times 314 = 1256 \text{ mm}^2$$

Step-1

To find the depth of neutral axis

$$0.36 f_{ck} b x_u + A_{sc} f_{sc} = 0.87 f_y A_{st}$$

$$\frac{d'}{d} = \frac{40}{400} = 0.1 \text{, more higher value } 0.1 \text{ may be adopted.}$$

$$f_{sc} = 217 \text{ N/mm}^2$$

Then

$$x_u = \frac{0.87 f_y A_{st} - A_{sc} f_{sc}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 250 \times 1256 - 402 \times 217}{0.36 \times 20 \times 230}$$

$$= 112.29$$

$$x_{u, \text{max}} = 0.53d$$

$$= 0.53 \times 400$$

$$= 212 \text{ mm}$$

$$x_u > x_{u, \text{max}}$$

The section is over reinforced.

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

$$= 0.36 \times 20 \times 230 \times 112.29 (400 - 0.42 \times 112.29)$$

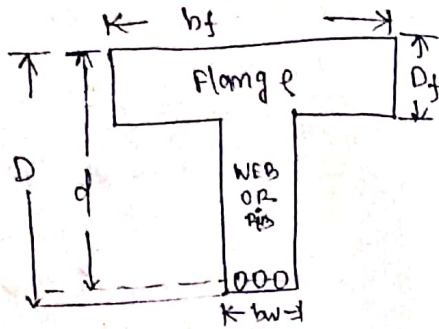
$$+ 402 \times 217 (400 - 40)$$

$$= 97015293.65$$

$$= 97.2 \text{ kNm}$$

10-1-04-09-12

Flanged OR T-Beam :- (LSM)



Position of N.A

Total compressive force in T-Beam.

$$F_{cc} = 0.36 f_{ck} b_f D_f$$

Total tensile force in T-Beam

$$F_{ts} = 0.87 f_y A_{st}$$

$F_{cc} > F_{ts}$ (N.A lies in flange)

$F_{cc} = F_{ts}$ (N.A lies on the bottom of flange)

$F_{cc} < F_{ts}$ (N.A lies on web)

The flange beam analysis & design are similar to doubly reinforced beam.

Case-1 Neutral axis lies in the flange ($x_u < D_f$)
when the N.A lies on the flange D_f will be replaced by x_u .

The compression zone area will become $b_f x_u$
 M_{ulimT} = moment resisting capacity for a flanged section.

For a single reinforced beam:

$$C = T$$

$$\Rightarrow 0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

For under reinforced section

$$\text{Lever arm } (z) = (d - 0.42 x_u)$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.36 f_{ck} b_f x_u (d - 0.42 x_u)$$

For over reinforced/balanced section

$$M_{ulimT} = 0.36 f_{ck} b_f x_{u,max} (d - 0.42 x_{u,max})$$

For a doubly reinforced flanged beam:

$$M_u - M_{ulimT} = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

$$A_{st1} = A_{st,lim}$$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y A_{st1}}$$

$$A_{st} = A_{st1} + A_{st2}$$

Problem:-

A T-beam of effective flange width 1200mm thickness of slab 100mm, width of rib 300mm & effective depth of 560mm is reinforced with 4-25mm dia bars. Calculate the factored moment of resistance. The materials are M_{20} grade concrete & Fe415 HYSD reinforcement.

Given data

$$b_f = 1200 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$d = 560 \text{ mm}$$

$$d_w = 560 - 100 = 460 \text{ mm}$$

$$b_w = 300 \text{ mm}$$

$$M_{20} = f_{ck} = 20 \text{ N/mm}^2$$

$$Fe 415 = f_{st} = 415 \text{ N/mm}^2$$

$$A_{st} = 4-25 \#$$

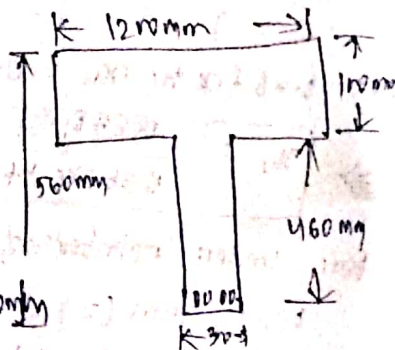
$$= 1963 \text{ mm}^2$$

$$f_{te} = 0.36 f_{ck} b_f D_f$$

$$= 0.36 \times 20 \times 1200 \times 100$$

$$= 864000 \text{ N}$$

$$= 864 \text{ kN}$$



$$f_{ts} = 0.87 f_y A_{st}$$

$$= 0.87 \times 415 \times 1963$$

$$= 708441.15 \text{ N}$$

$$= 709 \text{ kN}$$

$$f_{te} > f_{ts} \text{ (N-A lies in flange)}$$

for a singly reinforced section

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 1963}{0.36 \times 20 \times 1200} = 82.03 \text{ mm}$$

$$\text{lever arm (z)} = (d - 0.42 x_u)$$

$$= (560 - 0.42 \times 82.03)$$

$$= 525.51$$

Type of beam

$$x_{u \text{ max}} = 0.48 d \text{ (for Fe 415)}$$

$$= 0.48 \times 560$$

$$= 268.8 \text{ mm}$$

$$x_u < x_{u \text{ max}} \text{ (O.K.)}$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1963 (560 - 0.42 \times 82.03)$$

$$= 372 \text{ kNm}$$

10/11/2018

Problem

Given data

$$b_f = 130 \text{ mm}$$

$$d_f = 540 \text{ mm}$$

$$d_s = 100 \text{ mm}$$

$$d_w = 540 - 100 = 440 \text{ mm}$$

$$b_w = 80 \text{ mm}$$

$$m_{20} = f_{ck} = 20 \text{ N/mm}^2$$

$$f_{y415} = f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 4 \times 25 \#$$

$$= 1963 \text{ mm}^2$$

$$f_{tc} = 0.36 f_{ck} b_f d_f$$

$$= 0.36 \times 20 \times 130 \times 100$$

$$= 936 \text{ kN}$$

$$f_{ts} = 0.87 f_y A_{st}$$

$$= 0.87 \times 415 \times 1963$$

$$= 709 \text{ kN}$$

$f_{tc} > f_{ts}$ (N.A. lies in flange)

for singly reinforced section

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 415 \times 1963}{0.36 \times 20 \times 130}$$

$$= 76 \text{ mm}$$

$$\text{lever arm (z)} = (d - 0.42 x_u)$$

$$= (540 - 0.42 \times 76)$$

$$= 508.08 \text{ mm}$$

$$x_{u, \text{max}} = 0.48 d$$

$$= 0.48 \times 540$$

$$= 259.2 \text{ mm}$$

$$x_u < x_{u, \text{max}} \text{ (V.R.S.)}$$

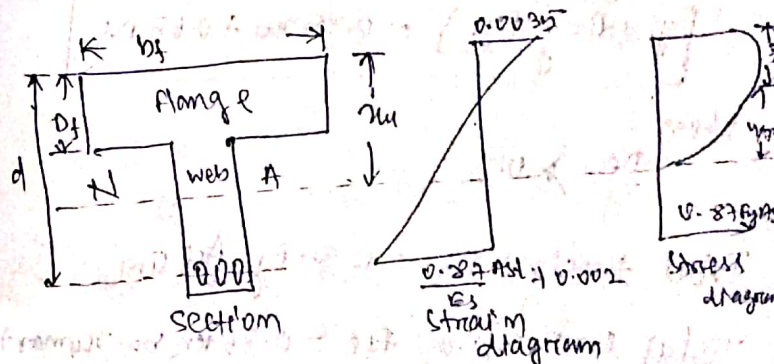
$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1963 (540 - 0.42 \times 76)$$

$$= 360 \text{ kNm}$$

Case-2

$f_{tc} < f_{ts}$ (N.A. lies on the web)



When mild steel is used the stress value in concrete will be uniform up to $3\epsilon_{yk}$.

Note:-

When the thickness of the flange exceeds $0.2d$ then the stress in flange will not be uniform when $\frac{D_f}{d} \leq 0.2$ then total compression in rectangular beam will be $(b_w \cdot d)$.

$$A_{fc} = 0.36 f_{ck} b_w \cdot x_u + 0.446 f_{ck} (b_f - b_w) x_u$$

$$M_u = 0.36 f_{ck} b_w \cdot x_u \cdot (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) x_u \cdot \left(d - \frac{x_u}{2}\right)$$

To find the steel area

$$A_{st_{lim}} = \frac{0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) x_u}{0.87 f_y}$$

NOTE:-

$$\text{If } \frac{D_f}{d} < 0.2$$

then D_f will be replaced by y_f

$$\text{yielding } (y_f) = 0.75 x_u + 0.65 D_f$$

When

$$\frac{D_f}{d} > 0.2$$

$$\text{Total tension } A_{st} = 0.87 f_y A_{st_{lim}}$$

$$\text{Total compression } A_{fc} = 0.36 f_{ck} b_w x_{u_{max}} + 0.446 f_{ck} (b_f - b_w) y_f$$

$$M_{u_{lim}} = 0.36 f_{ck} b_w x_{u_{max}} (d - 0.42 x_{u_{max}}) + 0.446 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2}\right)$$

$$A_{st_{lim}} = \frac{0.36 f_{ck} b_w x_{u_{max}} + 0.446 f_{ck} (b_f - b_w) y_f}{0.87 f_y}$$

Case-2

N.A lies in the web but the section is under reinforced.

→ Thus stress block for concrete show a uniform stress up to $\frac{3}{4} x_u$ and then parabolic shape beyond $\frac{3}{4} x_u$ up to $\frac{y_f}{2}$

$$D_f \leq \frac{3}{4} x_u$$

$$\text{Total compression} = 0.36 f_{ck} b_w x_{u_{max}} + f_{ck} \gamma (b_f - b_w) D_f$$

$$\text{Total tension} = 0.87 f_y A_{st_{lim}}$$

$$M_{u_{lim}} = 0.36 f_{ck} b_w \cdot x_u (d - 0.42 x_u) + 0.446 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

Total moment

$$M_u = M_{u1} + M_{u2}$$

$$A_{st} = A_{st1} + A_{st2}$$

$$A_{st2} = \frac{0.446 f_{ck} (b_f - b_w) D_f}{0.87 f_y}$$

$$M_{u1} = 0.36 f_{ck} b_w \cdot x_{u_{max}} (d - 0.42 x_{u_{max}})$$

$$M_{u2} = 0.446 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

$$\text{When } D_f > \frac{3}{4} x_u$$

$$\text{Total compression} = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_f$$

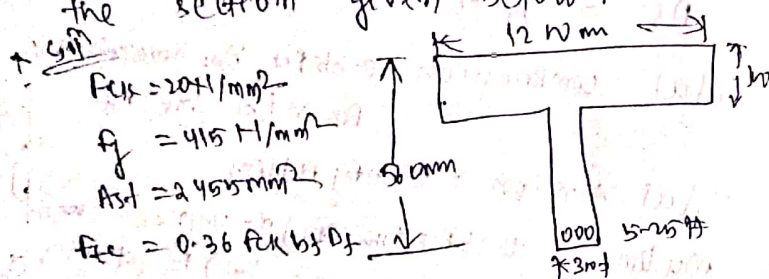
$$\text{Total tension} = 0.87 f_y A_{st}$$

$$M_u = 0.36 f_{ck} b_w x_u \left(d - 0.42 x_u \right) + 0.446 f_{ck} (b_f - b_w) y_s \left(d - \frac{y_s}{2} \right)$$

Problem

Find the moment of resistance for

the section given below.



$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 2455 \text{ mm}^2$$

$$f_{te} = 0.36 f_{ck} b_f D_f$$

$$= 0.36 \times 20 \times 1200 \times 100$$

$$= 864 \text{ kN}$$

$$f_{ts} = 0.87 f_y A_{st}$$

$$= 0.87 \times 415 \times 2455$$

$$= 886 \text{ kN}$$

$$f_{te} < f_{ts} \text{ (N.A. lies in web)}$$

$$\frac{D_f}{d} \geq 0.2 d$$

$$\frac{100}{500} \geq 0.2 \times 500$$

$$\Rightarrow 0.17 \geq 0.2$$

$$\text{Assume } D_f > \frac{3}{7} x_u$$

$$\text{Total compression} = 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y_s$$

$$y = 0.15 x_u + 0.65 D_f$$

$$\text{Total tension} = 0.87 f_y A_{st}$$

$$C = T \Rightarrow 0.36 f_{ck} b_w x_u + 0.446 f_{ck} (b_f - b_w) y = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 300 x_u + 0.446 \times 20 (1200 - 300) y = 0.87 \times 415 \times 2455$$

$$\Rightarrow 2160 x_u + 8028 y = 886377.75$$

$$\Rightarrow 2160 x_u + 8028 (0.15 x_u + 0.65 \times 100) = 886377.75$$

$$\Rightarrow 2160 x_u + 1204 x_u + 521820 = 886377.75 = 0$$

$$\Rightarrow 3364 x_u = 364557.75$$

$$\Rightarrow x_u = 108.37 \text{ mm}$$

$$D_f > \frac{3}{7} x_u$$

$$100 > \frac{3}{7} \times 108.37$$

$$100 > 46.44$$

$$x_{u \text{ max}} = 0.48 \times 500 = 240$$

$$x_u < x_{u \text{ max}} \text{ (U.R.S)}$$

$$y_s = 0.15 \times 108.37 + 0.65 \times 100$$

$$= 81.25$$

$$\begin{aligned}
 M_u &= 0.36 f_{ck} b_w x_{umax} \left(d - 0.42 x_{umax} \right) + \\
 &\quad 0.446 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right) \\
 &= 0.36 \times 20 \times 310 \times 108.32 \left(560 - 0.42 \times 108.32 \right) \\
 &\quad + 0.446 \times 20 \times (910 - 310) \times 81.25 \left(560 - \frac{81.25}{2} \right) \\
 &= 459.2 \text{ kNm}
 \end{aligned}$$

Q.

Given

$$b_f = 1200 \text{ mm}$$

$$D_f = 110 \text{ mm}$$

$$d = 560 \text{ mm}$$

$$d = 560 - 110 = 460 \text{ mm}$$

$$m_{20} = f_{ck} = 20 \text{ N/mm}^2$$

$$f_{c415} = f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 6.28 = 3694 \text{ mm}^2$$

$$f_{tc} = 0.36 f_{ck} b_f D_f$$

$$= 0.36 \times 20 \times 1200 \times 110$$

$$= 864 \text{ kN}$$

$$f_{ts} = 0.87 f_y A_{st}$$

$$= 0.87 \times 415 \times 3694$$

$$= 1333 \text{ kN}$$

$$f_{tc} < f_{ts} \text{ (N.A. lies on web)}$$

Chapter Shear, Bond Anchorage & Development Length

DI-22-09-12

Shear:-

The section of a structural member may be subjected to shear force due to bending, punching or torsion.

Flexural shear

The shear associated with the change in bending moment along the span is known as flexural shear.

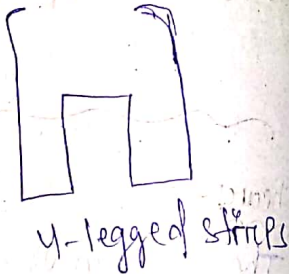
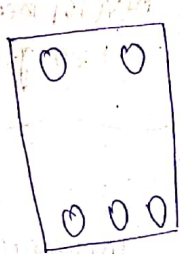
Punching shear:-

The shear associated with possibility of a punching of a thin member by a concentrated load is known as punching shear.



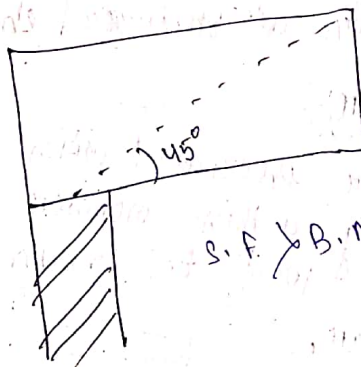
Torsional shear:-

The shear associated with torsion is known as torsional shear.



Types of shear failure:-

Diagonal shear failure

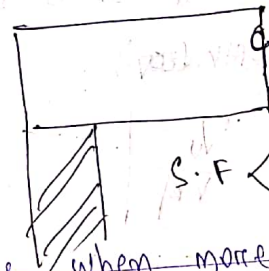


$S.F. > B.M$

(Page-72)

It occurs when a large shear force acts on the members and having subjected to less bending moment.

Flexural shear failure:-

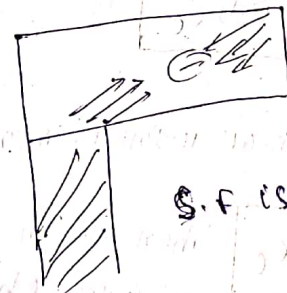


$S.F. < B.M$

It occurs when more bending moment

It occurs when the member is subjected to large B.M and less S.F.

Diagonal compression failure



$S.F. \text{ is max}$

(Page-73)

allowable shear stress / direct stress (τ_c)

It mainly depends on grade of concrete (f_{ck})
 i) Grade of concrete (f_{ck})
 ii) Percentage of steel (P_t) (Page-73)

ii) Nominal shear stress (τ_v)

The nominal shear stress in beam of uniform depth (d)

$$\tau_v = \frac{V_u}{bd}$$

where,

V_u = factored shear force

Case-1

No shear reinforcement

$$V_u < 0.5 \tau_c$$

Case-2

minimum shear reinforcement

$$\tau_v \leq \tau_c$$

Then design for minimum shear reinforcement

According to IS Clause no. 26.5.2

Page = 48

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

Here,

A_{sv} = Total c/s area of stirrup legged.

s_v = spacing of stirrup

b = breadth or width of beam

Case-3

shear reinforcement

$$\tau_v > \tau_c$$

$$V_{us} = V_u - V_c$$

$$V_{us} = \tau_v bd - \tau_c bd$$

$$V_{us} = (\tau_v - \tau_c) bd$$

$$\begin{aligned} V_u &= \tau_v bd \\ V_c &= \tau_c bd \end{aligned}$$

Here,

V_{us} = factored shear force in ^{residual} shear force

If this case comes then the reinforcement will be provided in the following forms

1. vertical stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

Page-73

Inclined stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sin \alpha + \cos \alpha)$$

Bentup Bars

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha$$

Q.

Design of vertical stirrups

Given data

$$\begin{aligned} b &= \\ d &= \\ V_u &= \\ A_{st} &= \end{aligned} \quad \begin{aligned} f_{ck} &= \\ f_y &= \end{aligned}$$

Step-1

To find the nominal shear stress

$$\tau_v = \frac{V_u}{bd}$$

Step-2

Calculate the design strength of concrete from f_t and f_{ck} table (Table no. - 19) page (72) by interpolation

At f_{ck}

x_1 mm y_1 mm
 x_2 mm y_2 mm

$$p_1 = \frac{100 A_{st}}{bd} = \text{value } x$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Step-3

check ' τ_v ' and ' τ_c '

If $\tau_v < \tau_c$ then calculate bare minimum shear reinforcement.

$$\frac{A_{sv}}{b.s_v} \geq \frac{0.4}{0.87 f_y}$$

~~Ass~~ stirrup size 6-8 mm dia bars

Assume A_{sv} = 6 to 8 mm dia bars

$$\left. \begin{aligned} s_v &= ? \\ s_v &\neq 300 \text{ mm} \\ s_v &\neq 0.75 d \end{aligned} \right\} \text{which, however is less}$$

Q: Design for shear if the width of the beam is 350 mm, effective depth 500 mm the factored shear force is 350 kN. The materials are M15 grade concrete & mild steel reinforcement. 4-25 mm dia bar are used.

Given data:-

$$b = 350 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$V_u = 350 \text{ kN}$$

$$f_{ck} = 15 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$A_{st} = 4-25 = 1963 \text{ mm}^2$$

Step-1

$$\tau_v = \frac{V_u}{b d}$$

$$\tau_v = \frac{350 \times 10^3 \text{ N}}{350 \times 500 \text{ mm}}$$

$$= 2 \text{ N/mm}^2$$

Step-2 To find the design shear strength

$$R_t = \frac{100 A_{st}}{b d} = \frac{100 \times 1963}{350 \times 500} = 1.12$$

ρ	1.1	f_{ck}	15
η_1	1.10	η	?
η_2	1.12	η_2	0.64
η_3	1.25		

$$\eta - \eta_1 = \frac{\eta_2 - \eta_1}{\eta_3 - \eta_1} (\rho - \rho_1)$$

$$\eta - 0.60 = \frac{0.64 - 0.60}{1.25 - 1.10} (1.12 - 1.10)$$

$$\eta - 0.60 = 0.19$$

$$\eta = 0.19 + 0.60 = 0.79 \approx 0.62$$

$$= 0.62 \text{ N/mm}^2 = \tau_c$$

Step-3

$$\tau_v < \tau_c$$

$$2 \text{ N/mm}^2 < 0.62 \text{ N/mm}^2$$

Design shear reinforcement is not required.

Design for shear reinforcement

$$\begin{aligned} V_{us} &= V_u - V_c \\ &= (\tau_v - \tau_c) b d \\ &= (2 - 0.67) 250 \times 520 \\ &= 241500 \text{ N} \\ &= 241.5 \text{ kN} \end{aligned}$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

Assume 2 legged 8 mm dia stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$A_{sv} = 100 \text{ mm}^2$$

$$\begin{aligned} (\text{Spacing}) S_v &= \frac{0.87 \times 250 \times 100 \times 520}{241.5 \times 10^3} \\ &= 45 \text{ mm} \end{aligned}$$

$$S_v = 100 \text{ mm}$$

$$S_v = 300 \text{ mm}$$

$$S_v = 375 \text{ mm}$$

$$2L \ 8\# @ 100 \text{ mm c/c}$$

DA - 25-09-18

Q. A beam with 432 mm bars as main tensile reinforcement has 2 of its main bars symmetrically bent at the end of the beam at 45° . Find the stirrups required for resistance against shear failure at the end of the beam. Assume $b = 250 \text{ mm}$, $d = 520 \text{ mm}$ and the materials are M25 grade concrete and Fe 415 HYSD reinforcement.

Given data

$$b = 250 \text{ mm}$$

$$d = 520 \text{ mm}$$

$$V_u = 400 \text{ kN}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 32^2 = 3216 \text{ mm}^2$$

Step-1 To find the Nominal Shear Stress

$$\tau_v = \frac{V_u}{b d}$$

$$= \frac{350 \times 410 \times 10^3}{350 \times 550} = 2.0779 \text{ MPa}$$

Step-2 To find the design Shear strength

$$R_L = \frac{100 A_c}{b d} = \frac{100 \times 3016}{350 \times 550} = 1.67$$

R_L	F_{ck}
$\alpha_1 1.50$	$y_1 0.74$
$\alpha 1.67$	$y = 0.74$
$\alpha_2 1.75$	$y_2 0.78$

$$y - y_1 = \frac{y_2 - y_1}{\alpha_2 - \alpha_1} (\alpha - \alpha_1)$$

$$y - 0.74 = \frac{0.78 - 0.74}{1.75 - 1.50} (1.67 - 1.50)$$

$$y = 0.74 + 0.027 = 0.767$$

$$y = 0.74 + 0.027 = 0.767 \text{ N/mm}^2 = \tau_c$$

Step-3 $\tau_v > \tau_c$

$$2.0779 > 0.76$$

design for shear reinforcement

$$V_{us} = V_u - V_c$$

$$= (\tau_v - \tau_c) b d$$

$$= (2.0779 - 0.76) 350 \times 550$$

$$= 250.78 \Rightarrow 251 \text{ kN}$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin \alpha + \cos \alpha)$$

Assume 2 legs of 8mm dia. stirrup

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} (\sin \alpha + \cos \alpha)$$

$$= \frac{0.87 \times 415 \times 100 \times 550}{61251 \times 10^3} (\sin 50^\circ + \cos 50^\circ)$$

$$= \cancel{89.27} \text{ mm} \quad 111.88$$

checked

$$S_v = 111.88 \approx 112$$

$$S_v = 89 \text{ mm}$$

$$S_v = 0.45 \times d$$

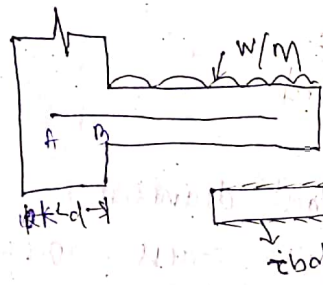
$$= 0.45 \times 550$$

$$= 247.5$$

provide 2 legged 8 mm ϕ inclined stirrups @ 112 mm c/c

2L 8# @ 112 mm c/c

Bond anchorage



Here

τ_{bd} = bond stress

L_d = development length

Bond stress

In R.C.C designed bond refers to the adhesion or the shear stress that occurs between concrete and steel in a loaded member.

→ It enables the two materials to act together.

Development length:-

The length of the bar necessary to develop the full strength of the section is known as development

length.

According to clause number (26.2.1.1)
page-42

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

where

ϕ = nominal diameter

σ_s = Tensile stress - $0.87 f_y$

τ_{bd} = Designed bond stress (26.2.1.1)

NOTE

For deformed bars (simply supported beam)
bond stress is increase by 60%.

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd} (1.6)}$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd} (1.6)} \quad \text{(for simply supported beam)}$$

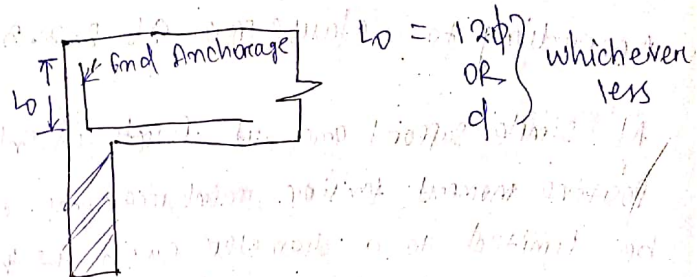
For cantilever beam

The bond stress is increased by 25%.

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd} (1.25)}$$

END Anchorage of bar

For continuous beam "L₀"



For simply supported

$$L_0 = 8 \phi$$

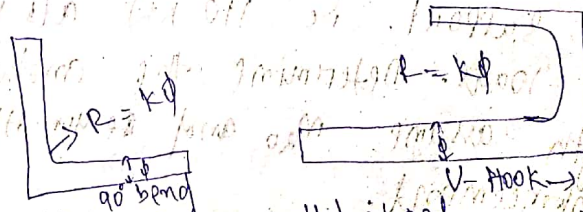
Anchorage length of Hooks and bend

According to clause number 26.2.2.1

For standard Hook = 16ϕ

for 90° bend $L_0 = 8 \phi$

for 45° bend $L_0 = 4 \phi$



$k = 2$ for mild steel

4 for HYSD reinforcement

checked for Development length for tension

According to clause no. 26.2.3.3

At simple support and at points of inflection positive moment tension reinforcement shall be limited to a diameter such as that L_d should not be more than $(\frac{m_1}{V} + L_0)$

$$\frac{m_1}{V} + L_0$$

For simply supported beam

$$L_d \geq 1.3 \frac{m_1}{V} + L_0$$

Q. A simply supported beam of width 85 cm and overall depth 550 mm and 2-20 mm bars is used. At the centre of the support is 110 kN at work ing loads. Determine the anchorage length assume m_{20} and $f_y = 415$ MPa reinforcement.

solⁿ Given data

$$b = 250 \text{ cm} = 250 \text{ mm}$$

$$d = 500 \text{ cm} = 500 \text{ mm} \quad C.C. = 50 \text{ mm}$$

$$d = 500 - 50 = 450 \text{ mm}$$

$$A_{st} = 2-20$$

$$= 2 \times \frac{\pi}{4} \times 20^2 = 628 \text{ mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd} (1.6)}$$

$$= \frac{\phi \times 361}{4 \times 1.2 \times 1.6}$$

$$L_d = 47 \phi$$

$$\sigma_s = 0.87 \times f_y = 0.87 \times 415 = 361 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \sqrt{f_{ck}}$$

step-1 check for Development length

$$L_d \geq 1.3 \frac{m_1}{V} + L_0 \quad (\text{for s/s beam})$$

$$m_1 = 0.87 f_y A_{st} (d = 0.42 m)$$

$$\tau_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250} = 126 \text{ MPa}$$

$$\mu_{max} = 0.48d$$

$$= 0.48 \times 450 = 216 \text{ mm}$$

$$\mu_u < \mu_{max} (V.R.S)$$

$$M_1 = 0.87 \times 415 \times 688 (450 - 0.42 \times 126)$$

$$= 90 \text{ kNm}$$

$$V = 1.5 \times 110 = 165 \text{ kN}$$

$$L_0 = 8\phi$$

$$= 8 \times 20$$

$$= 160$$

We know

$$L_d \geq 1.3 \frac{M_1}{V} + L_0$$

$$L_d = 1.3 \frac{M_1}{V} + L_0$$

$$47\phi = \frac{1.3 \times 90 \times 10^6}{165 \times 10^3} + 8\phi$$

$$\Rightarrow 47\phi - 8\phi = 709 \text{ mm}$$

$$\Rightarrow 39\phi = 709 \text{ mm}$$

$$\Rightarrow \phi = \frac{709}{39} \text{ mm} = 18.17 \text{ mm}$$

$$\approx 18.17 \text{ mm} \approx 20 \text{ mm}$$

Then

$$L_d = 47\phi$$

$$= 47 \times 20$$

$$= 940 \text{ mm}$$

$$= 0.94 \text{ m}$$

26.09.18

Problem shear, bond anchorage & development length

A beam with 4-30 mm bar as main tensile reinforcement has two ob lts bar symmetrically bend as 60° bind the stirrup required against shear failure. If the factored shear force at the critical section is 450 kN. Assume $b = 300$, $d = 500$, $f_{ck} = 20 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$.

Sol Given

$$b = 300 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 30^2 = 2827 \text{ mm}^2$$

$$V_u = 450 \text{ kN}$$

$$\phi = 60^\circ$$

$$\text{Step-1} \quad \tau_v = \frac{V_u}{bd} = \frac{450 \times 10^3}{300 \times 500} = 3 \text{ N/mm}^2$$

$$\text{Step-2} \quad \rho_t = \frac{100 A_{st}}{bd} = \frac{100 \times 2827}{300 \times 500} = 1.88$$

$$\begin{aligned} \frac{\rho_t}{f_{ck}} & \\ x_1 = 1.75 & \quad y_1 = 0.75 \\ x = 1.88 & \quad y = \\ x_2 = 2.0 & \quad y_2 = 0.79 \end{aligned}$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 0.75 = \frac{(0.79 - 0.75)}{(2.0 - 1.75)} (1.88 - 1.75)$$

$$= 0.02$$

$$y = 0.75 + 0.02 = 0.77$$

$$y = 0.77 = \tau_c$$

Check

$$\tau_v > \tau_c$$

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

$$V_{us} = (\tau_v - \tau_c) b d$$

$$= (3 - 0.77) \times 300 \times 500 = 334.5 \text{ kN}$$

Assume 2-legged 8mm bars stirrups

$$A_{sv} = 2 \times \pi \times 8^2$$

$$= 100 \text{ mm}^2$$

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} (\sin \alpha + \cos \alpha)$$

$$= \frac{0.87 \times 415 \times 100 \times 500}{334.5 \times 10^3} (\sin 60 + \cos 60)$$

$$= 73.74 \text{ mm}$$

$$s_v = 73.74 \approx 80 \text{ mm}$$

$$s_v = 80 \text{ mm}$$

$$s_v = 0.75 d$$

$$= 0.75 \times 500$$

$$= 375 \text{ mm}$$

Provide 2-legged 8mm ϕ inclined stirrup

2L 8# @ 80mm c/c

Chapter -

DT - 27.09.18

COLUMN

A compression member whose length is 3 times its least dimension.

Classification of column:-

- 1) According to shape
- 2) According to slenderness ratio
- 3) According to type of loading

According to shape:-

- i) It can be divided into
- ii) Rectangular column
- iii) Square column
- iv) Circular column

According to slenderness ratio:-

- i) Long column
- ii) Short column

According to type of loading:-

- i) Axially loaded
- ii) Axial and uniaxial loaded
- iii) Axial and Biaxial loaded

$$\frac{l}{b_{\text{min}}} + \frac{d}{b_{\text{min}}} = 20 \text{ mm}$$

$$e_{\text{min}} = 20 \text{ mm}$$

e = eccentricity

 eccentricity

Perce = 42, 25.4
43, 26.2-2.1
Anchorage bar length

Design steps for Vertical Reinforcement

Step-1

To check $e_{\text{min}} < 0.05D$
or
 $e_{\text{min}} \leq 20 \text{ mm}$

Step-2

To find the factored load

$$\text{factured load} = 1.5 \times \text{Actual load} (P_u)$$

Step-3

To find the gross sectional area

Ag

$$A_c = \text{concrete area} = 0.992 A_g$$

$$A_{sc} = 0.008 A_g \text{ (concrete in steel area)}$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_{yk} A_{sc}$$

Step-4

To find $A_{sc \text{ required}} = 0.008 A_{g \text{ required}}$

Step-5

To find the number of steel or bars

$$n = \frac{(A_{sc})_{req}}{\pi/4 (d)^2} \quad (\text{where } d = 16, 20, 25 \text{ mm})$$

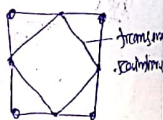
Step-6

To find the dia of transverse reinforcement

$$= \phi_{tr} < \frac{\phi}{4}$$

or

$$\phi_{tr} = 6 \text{ mm to } 8 \text{ mm}$$



Step-7

To find the Pitch

$$\left. \begin{array}{l} P = D \\ P < 16 \phi \end{array} \right\} \text{whichever is less}$$

$$P = 300$$

Q.

A short RCC column is to carry a factored load of 1900 kN. If the column is to be a square then design the column. Assume $e_{min} = 0.05D$ the materials are M20 grade concrete and mild steel reinforcement.

Sol

Given data

$$P_u = 1900 \text{ kN}$$

$$e_{min} = 0.05D$$

$$20 = 0.05D$$

$$\Rightarrow D = \frac{20}{0.05} = \frac{e_{min}}{0.05}$$

$$\Rightarrow D = 400 \text{ mm}$$

400 x 400 for square column

$f_c = 20 \text{ N/mm}^2 = f_y$ } square column
 $f_{ck} = 20 \text{ N/mm}^2$

Step-2

To find the gross-sectional area

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$= 0.4 \times 20 \times 0.992 A_g + 0.67 \times 250 \times 0.008 A_g$$

$$= 7.936 A_g + 1.34 A_g$$

$$= A_g (7.936 + 1.34)$$

$$P_u = A_g \cdot 9.276$$

$$A_g = \frac{P_u}{9.276} = \frac{1900 \text{ kN}}{9.276} = \frac{1900 \times 10^3}{9.276}$$

$$= 204830 \text{ mm}^2$$

$$A_{greq} = 204830 \text{ mm}^2$$

$$D^2 = 204830$$

$$D = 452 \text{ mm}$$

$$\Rightarrow 450 \text{ mm}$$

$$A_{greq} = 450 \times 450 = 202500 \text{ mm}^2$$

A_{sc} required

$$\begin{aligned} A_e &= 0.992 A_{greq} \\ &= 0.992 \times 202500 \\ &= 200880 \end{aligned}$$

Step 3 To find the $A_{sc req}$

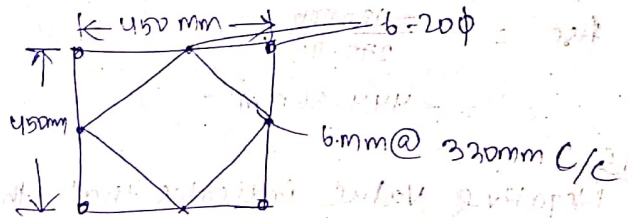
$$\begin{aligned} &= 0.008 A_{greq} \\ &= 0.008 \times 202500 \\ &= 1620 \text{ mm}^2 \end{aligned}$$

Step 4 To find the no. of rods

$$\begin{aligned} n &= \frac{(A_{sc req})}{\pi/4 (d)^2} = \frac{(1620)}{\pi/4 \times 20^2} \\ &= 5.15 \approx 6 \text{ no.} \end{aligned}$$

Step 5 Transverse reinforcement

$$\begin{aligned} &= \phi_{tr} \leq \frac{\phi}{4} \\ &= \phi_{tr} \leq \frac{20}{4} = 5 \\ &\approx \phi_{tr} \leq 6 \end{aligned}$$



Q. A short R.C.C column of size 450x450 has to carry on factored axial load of 1500 kN. Assume $e_{min} < 0.5D$. Design the column using M20 grade concrete and Fe 415 HYSD reinforcement.

Solⁿ Given data

$$P_u = 1500 \text{ kN}$$

$$M_{20} = f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$D = 450 \text{ mm}$$

$$A_c = A_g - A_{sc}$$

$$= 450 \times 450 - A_{sc}$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$= 0.4 \times 20 \times (450 \times 450 - A_{sc}) + 0.67 \times 415 A_{sc}$$

$$\Rightarrow 1500 \times 10^3 = 1620 \times 10^3 - 8 A_{sc} + 278.05 A_{sc}$$

$$\Rightarrow 270.05 A_{sc} = 1500 \times 10^3 - 1620000$$

$$\Rightarrow 270.05 A_{sc} = 120000$$

$$\Rightarrow A_{sc} = \frac{125000}{270.05} = -444.36 \text{ mm}^2$$

Note Negative value indicates that there is no need of providing reinforcement. However, minimum reinforcement should be provided.

Area of concrete required for direct load

$$A_c = \frac{P_u}{0.4 f_{ck}} = \frac{1500 \times 10^3}{0.4 \times 20} = 187500 \text{ mm}^2$$

$$A_{sc_{req}} = 0.008 A_c = 0.008 \times 187500 = 1500 \text{ mm}^2$$

To find the no. of rods

$$n = \frac{A_{sc_{req}}}{\pi/4 (d_r)^2} = \frac{1500}{\pi/4 \times 16^2} = 7.4 \approx 8 \text{ mm}$$

$$A_{sc_{prov}} = n \times \pi/4 \times d_r^2 = 8 \times \pi/4 \times 16^2 = 1608 \text{ mm}^2$$

transverse reinforcement

$$\phi_{tr} < \frac{\phi}{4}$$

$$\phi_{tr} < \frac{16}{4}$$

$$\phi_{tr} < 4$$

$$\phi_{tr} = 6$$

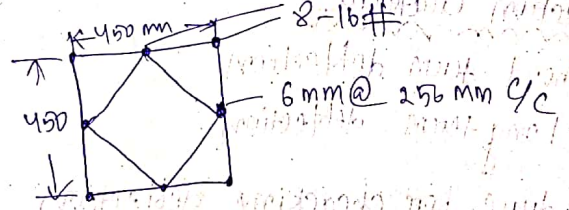
To find the pitch

$$P = 300 \text{ mm}$$

$$P = D = 450 \text{ mm}$$

$$P < 16 \phi$$

$$\Rightarrow P = 16 \times 16 = 256 \approx 250 \text{ mm}$$



Q. Design a circular short column to carry an axial working load.

SLAB

one way slab

Two way slab

where the depth is much smaller than its span and width.

Limit state of serviceability due to deflection:

Deflection Criteria:-

- short term deflection
- Long term deflection

Procedure for checking deflection

Step-1 Choose basic $\frac{\text{span}}{d}$ ratio as per code.

for cantilever beam basic $\frac{\text{span}}{d} = 7$

for SSB = 20

for continuous beam = 26

Step-2 Calculate modification factor from graph given in code book. (Page 38 & 39)

i.e. from P_t and f_s graph.

$$f_s = \frac{0.58 f_y A_{streq}}{A_{stprov}}$$

Service steel stress = f_s

Step-3

Calculate allowable $\frac{\text{span}}{d}$ ratio = $\frac{\text{Basic span}}{m_s}$

m_s = modification factor

Step-4

Calculate actual span ratio

Step-5

$$\frac{\text{Actual span}}{d} < \frac{\text{Allowable span}}{d}$$

Q. A SSB of span = 12m and effective depth 800mm is provided with 1.6% of tension reinforcement the area of tension steel required by calculation is only 1.25% check the deflection of the slab ok or not.

Solⁿ Given data

$$L = 12\text{m}$$

$$d = 750\text{mm}$$

$$A_{st, req} = 1.25\%$$

$$P_t = 1.25$$

Step-1 Basic span $= 20$

$$= \frac{20 \times 10^3}{12} = 16.67\text{m}$$

Step-2

$$F.S = 0.58 \times 500 \times \frac{1.25}{1.6}$$
$$= 226.5\text{mm}$$

$$P_t = \frac{100 A_{st, req}}{d} = \frac{100 \times 500}{12}$$

$$M_f = 1.09\text{mm}$$

$$\text{allowable} = \text{Basic span} \times M_f$$

$$= 16.67 \times 1.09$$

$$= 18.17\text{m}$$

$$\text{Actual span} = \frac{\text{actual}}{d} = \frac{12 \times 10^3}{750}$$
$$= 16\text{m}$$

Slab

for beam

$$d > b$$

for slab

$$b \geq d \text{ or } d \leq L$$

⇒ slabs are the plate elements forming floor and roofs of a building carrying VDL primarily by flexure.

⇒ In slab depth each much smaller than its span and width.

⇒ A slab may be supported by beams or walls and may be used as the change or a T-beam or L-beam.

⇒ It may be simply supported or continuous / depending upon the supporting condition.

Classification of slab:-

1) one-way slab

⇒ spacing in one direction only

2) Two way slab

⇒ spacing in two direction

- 3) Circular slab
- 4) Flat slab
- 5) Grid floor slab

1) one way slab

$\frac{L_y}{L_x} \geq 2$ then it is one way slab

Analysis of one way slab:-

Calculation of effective span

For simply supported slab

$L = \text{clear span} + \text{width of the}$

Support.

For continuous slab its width of the support $\leq \frac{L_c}{12}$

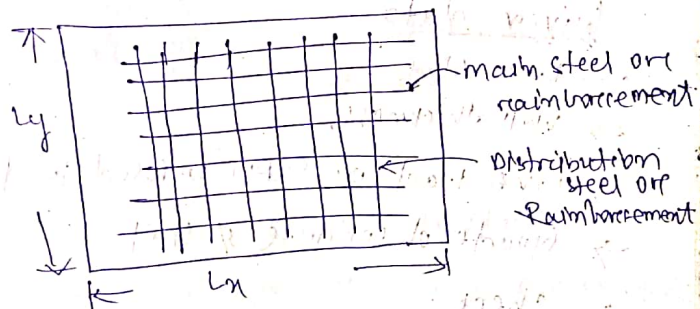
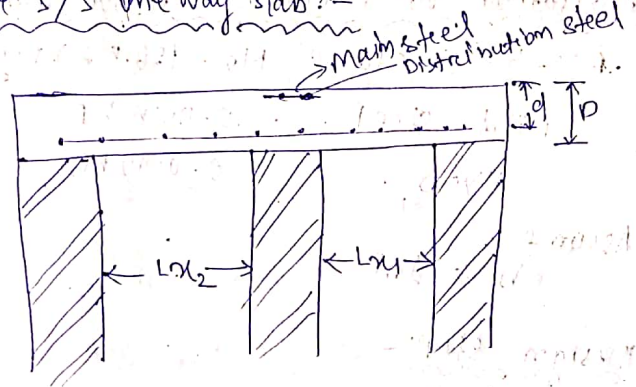
$L_c = \text{clear span}$

$L = L_c + d$

Calculation of Reinforcement Required



For s/s one way slab:-



Function of Distribution Steel:-

→ It resists the temperature stresses and shrinkage stress.

→ It keeps the main reinforcement in position and distribute the concentrated load and non uniform load through out the slab.

Calculation of minimum Reinforcement:-

Acc. to clause No. (26.5.2) Page-48

$$\text{mild steel} = 0.0015 b D$$

$$\text{HYSD} = 0.0012 b D$$

Assume $b = 1\text{m}$

Design step:-

Given data

- Room size
- Wall thickness,
- Live load or super imposed load
- Grade of concrete & steel
- check $\frac{L_y}{L_n} \geq 2$ (for one-way slab)

Step-1 Calculation of depth of slab for a one-way slab.

- for s.s beam $\frac{\text{clear span}}{d} = 20$
- for continuous beam $\frac{\text{clear span}}{d} = 26$
- for cantilever beam $\frac{\text{clear span}}{d} = 7$

for s.s.b $\frac{\text{clear span}}{d} = 20$

$$\Rightarrow d = \frac{\text{clear span}}{20}$$

$$D = d + c.c + \phi \quad (\phi < 12\text{mm})$$

Step-2 Calculation of Total load (T_L)

Assume 1m width of slab
 $b = 1\text{m}$

$$\text{Total load} = \text{Dead load} + \text{Live load} + \text{Floor finish (AF)}$$

$$\text{Dead load} = 25 b D$$

$$\text{Floor finish} = 1 \times 1 \times N/\text{m}^2 \text{ (for floor)}$$

$$\text{Factored total load} = 1.5 \times \text{total load}$$

Step-3 Calculation of effective span

$$\text{Effective span} = \text{shorter span of the slab (L}_n\text{)} + \text{clear span}$$

$$E_s = d + L_y$$

Step-4 Calculation of max^m B.M & S.F

$$\text{max}^m \text{ B.M for s.s.b} = \frac{wL^2}{8}$$

$$\text{max}^m \text{ S.F for s.s.b} = \frac{wL}{2} \quad [L = \text{clear span}]$$

Step-5 Calculation of effective depth required for balance section

$$M_u = 0.148 f_{ck} b d^2 \text{ (for } f_{ck} = 20 \text{)}$$

$$M_u = 0.138 f_{ck} b d^2 \text{ (for } f_{ck} = 15 \text{)}$$

$$M_u = 0.133 f_{ck} b d^2 \text{ (for } f_{ck} = 50 \text{)}$$

Step-6 Design for Flexure

$$M_u = M.O.R$$

$$M.O.R = 0.87 f_y A_{st} \left(d - \frac{A_{st} f_y}{f_{ck} b} \right)$$

a) A_{st}

b) Calculate no. of bars $\nless(A_{st,prov})$

c) Calculate spacing

d) Half of the bars are bent up at a distance of 0.1L

$$A_{st,prov} = 0.5 A_{st}$$

e) Calculate distribution steel

f) Calculate spacing for distribution steel

$$A_{st} = \frac{1}{4} (d^2) \times \frac{b}{\text{spacing}}$$

Step-7 Check for Shear

$$a) \tau_v = \frac{V_u}{b d} \quad (b = 1 \text{ m})$$

$$b) \tau_c = \frac{50 A_{st}}{b d}$$

$$P_t = \frac{100 A_{st}}{b d} \Rightarrow 100 \left(\frac{0.5 A_{st}}{b d} \right)$$

c) calculate τ_c & τ_{cr}

d) check $\tau_v < \tau_c$ (safe or ok)

Step-8 Check for development length

$$L_d \leq 3.3 \frac{m_1}{V} + L_0$$

$$a) L_0 = 8 \phi$$

$$\& L_0 = 12 \phi$$

$$b) L_d = \frac{p_s}{4 \tau_{bd}}$$

$$c) m_1 > 0.87 f_y (0.5 A_{st}) / \left(1 - \frac{0.5 A_{st}}{A_{st,prov}} \right)$$

Step-9 Check for deflection by empirical meth basic $\frac{\text{Span}}{d}$ ratio

Step-10 Detailing of reinforcement

Q. Design a s/s slab for a room size of 8×3.5 m clear in size. The live load is 5 kN/m^2 use M15 grade concrete and Fe415 HYSD reinforcement.

Sol Given data

Room size = 8×3.5 m

Live load (L.L) = 5 kN/m^2

$f_{ck} = 15 \text{ N/mm}^2$, $L_y = 8$ m

$f_y = 415 \text{ N/mm}^2$, $L_x = 3.5$ m

Check

$$\frac{L_y}{L_x} \geq 2$$

$$\Rightarrow \frac{8}{3.5} \geq 2 \quad (\text{Design one-way slab})$$

$$\Rightarrow 2.28 \geq 2$$

Step-1

$$\text{For c.s.b} = \frac{\text{Clear span}}{d} = 20$$

$$\Rightarrow d = \frac{\text{Clear span}}{20}$$

$$\Rightarrow \frac{3500}{20} = 175 \text{ mm}$$

$$D = d + c.c + \frac{\phi}{2} \quad (\phi \neq 12 \text{ mm})$$

$$= 175 \text{ mm} + 15 \text{ mm} + \frac{10}{2} \text{ mm}$$

$$= 195 \text{ mm} \approx 0.195 \text{ m}$$

Step-2

$$T.L = L.L + D.L + F.F$$

$$L.L = \text{Given} \quad D.L = 25 b D$$

$$D.L = 25 \times 1 \times 0.195 = \text{KN/m}^2$$

$$= 4.875 \text{ KN/m}^2$$

$$T.L = 5 + 4.875$$

$$= 9.875 \text{ KN/m}^2$$

$$\text{Factored T.L} = 1.5 \times T.L$$

$$= 1.5 \times 9.875 = 14.81 \text{ KN/m}^2$$

Step-3

Effective span

$$L_{eff} = L_x + d$$

$$= 3.5 + 0.175$$

$$= 3.675 \text{ m}$$

Step-4

max^m bending moment in c.s.b

$$M_u = \frac{w_u l^2}{8}$$

$$= \frac{w_u L_{eff}^2}{8} \quad (\text{effective span})$$

$$M_u = \frac{14.81 \times (2.675)^2}{8}$$

$$= 24.48 \text{ kNm}$$

$$\text{max}^m \text{ s.f. } (V_u) = \frac{W_u L_y}{2}$$

$$= \frac{14.81 \times 3.5}{2}$$

$$= 25.92 \text{ kN}$$

Step 5 Calculate bar effective depth

$$M_u = 0.138 f_{ck} b d^2 \quad (\text{for } f_{ck} 415)$$

$$\Rightarrow 25 \times 10^6 = 0.138 \times 15 \times 1000 \times d^2$$

$$\Rightarrow d^2 = \frac{25 \times 10^6}{0.138 \times 15 \times 1000}$$

$$\Rightarrow d^2 = \sqrt{12077.29} = 109.89 \approx 110$$

$$\text{Assum } d = 150 \text{ mm}$$

$$D = d + c.c + \frac{d}{2}$$

$$= 150 + 15 + \frac{150}{2} = 170 \text{ mm}$$

Step 6 Design bar flexure

$$M_u = M.D.R$$

$$M_u = 0.87 f_y A_{st} \cdot d \left(d - \frac{A_{st} \cdot f_y}{f_{ck} b d^2} \right)$$

$$25 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \left(150 - \frac{A_{st} \times 415}{15 \times 1000 \times 150} \right)$$

$$\Rightarrow 25 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \left(1 - \frac{A_{st} \cdot 415}{15 \times 1000 \times 150} \right)$$

$$\Rightarrow 25 \times 10^6 = \dots$$

$$A_{st \text{ req}} = 509 \text{ mm}^2$$

b) no. of bars

$$n = \frac{A_{st \text{ req}}}{\pi/4 d^2}$$

$$= \frac{509}{\pi/4 \times 10^2} = \frac{509 \times 4}{314} = 6.48 \approx 7$$

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{st \text{ req}}}$$

$$= \frac{\pi/4 \times 10^2 \times 1000}{509}$$

$$= 154 \text{ c/c}$$

$$A_{st \text{ Prov}} = \pi/4 d^2 \cdot \frac{b}{\text{Spacing}}$$

$$= \pi/4 \times 10^2 \times \frac{1000}{150}$$

$$= 522 \text{ mm}$$

d) $0.12 \text{ m} = 0.11 \times 3.645 \times 10^{-3} = 0.401 \text{ m}$
 $0.3645 \times 10^{-3} = 0.3645 \text{ m}$

0.5% Ast prov $\rightarrow$ Bend

$$523 \times 0.5 = 261.5 \text{ mm}^2$$

$\rightarrow$ Distribution steel $= 0.0012 b d$
 $= 0.0012 \times 1000 \times 170$
 $= 204 \text{ mm}^2$

Spacing $= \frac{\text{Area of one bar} \times b}{204}$
 $= \frac{74 \times 6^2 \times 1000}{204} = 140 \text{ mm}$

no. of bars $n = \frac{74 \times 6^2}{204} = 2$

Step 7 check for shear

$$\tau_v = \frac{V_u}{b d} = \frac{25.92}{1000 \times 150}$$

$$\tau_v = 0.1724 / \text{mm}^2$$

$$\tau_c = \frac{50 \text{ Ast prov}}{1000 \times 523} = 0.174$$

$x_1 = 0.15$ $y_1 = 0.28$
 $x = 0.144$ $y =$
 $x_2 = 0.25$ $y_2 = 0.35$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 0.28 = \frac{0.35 - 0.28}{0.25 - 0.15} (0.25 - 0.15)$$

$$y = 0.294$$

Then $\tau_c = 1.2 \tau_c \times 0.29$

$\tau_v \leq \tau_c$ (The slab is safe in shear)

Step 8

a) $L_o = 8 \phi$

b) $L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{\phi \times 415 \times 0.87}{4 \times 1.6}$

$$= \frac{361.5 \phi}{6.4} = 56.4$$

c) $M_1 = 0.87 f_y (0.5 \text{ Ast}) d (1 - \frac{0.5 \text{ Ast} f_y}{f_{ck} b d})$

$$M_1 = 2013.5 \text{ kNm} = 13.5 \times 10^6 = 13500 \text{ Nm}$$

$$M_u \leq 25.92 \text{ kNm} = 25.92 \times 10^3 = 25920 \text{ Nm}$$

$$L_d = \frac{1.3 m_1}{\gamma_u} + L_0$$

$$= \frac{13 \times 13.5 \times 10^6}{25.92 \times 10^3} + 8\phi$$

$$56\phi = 8\phi$$

$$48\phi = 677$$

$$\phi = \frac{677}{48} = 14 \text{ (OK)}$$

step-9 check bar deflection

$$f_s = \frac{0.58 f_y A_{st, req}}{A_{st, prov}}$$

$$= \frac{0.58 \times 415 \times 509}{523}$$

$$= 235 \text{ mm}$$

$$d_l = \frac{100 A_{st, prov}}{b d} = \frac{100 \times 523}{1000 \times 150}$$

$$= 0.35$$

Allowable span $\therefore \leq \text{Basic span} \times M-F$

$$d_l \times 20 \times 1.4 = 28 \text{ mm}$$

$$\text{Allowable span} = 28 \text{ mm}$$

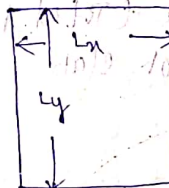
$$\text{Actual span} = \frac{2.5 \times 10^3}{150}$$

$$= 23.33 \text{ mm}$$

$$\frac{\text{Actual span}}{d_l} < \frac{\text{allowable span (OK)}}{d_l}$$

$$d_l = 31.10.18$$

Two-way slab:-



$\rightarrow$ If $L_y/L_x \leq 2$ then, design for Two-way slab.

$\rightarrow$ The slab is to be supported 4-edge.

* In simply supported slab the corners are not held down.

* Restrained slab on which the corners are held down.

$\rightarrow$ Design of two-way slab (SS) which corners are not held down.

Data given

- i) Room size
- ii) Live load
- iii) Grade of steel and concrete

$L_y/L_x \leq 2$
Assume overall thickness or depth of slab = 150 to 180 mm.

$$d_{eff} = D - c.c. - \frac{\phi}{2}$$

Step-1 Calculation of T.L (Total load)

Assume 1m width of slab

$$b = 1m$$

$$D.L = 25bD$$

$$\text{Floor finish} = 1 \times 1 = \text{KN/m}^2$$

$$T.L = D.L + F.F$$

$$\text{Factored total load } (W_u) = 1.5 \times T.L$$

Step-2 Calculation of effective span

$$E_s = L_x + d_{eff}$$

or

$L_x + \text{width of support}$

[whichever is less]

Step-3 Calculation of factored moment

$$M_x = \frac{W_u L_x^2}{8}$$

$$M_y = \frac{W_u (L_{eff})^2}{8}$$

M_x & M_y are bending moments

Step-4

Calculation of effective depth required for balanced design

$$m_x = 0.138 \text{ for } b/d^2 \text{ (for Fe 415)}$$

$$m_x = 0.143 \text{ for } b/d^2 \text{ (Fe 250)}$$

$$= 0.133 \text{ for } b/d^2 \text{ (Fe 500)}$$

$$d_{req} = \sqrt{\frac{M_x}{0.138 \text{ for } b}}$$

$$D = d_{req} + c.c. + \frac{\phi}{2}$$

$$d_{short span} = D - \frac{\phi}{2} - c.c.$$

$$d_{long span} = d_{short span} - \phi$$

Step-5 Calculation of Ast

a) Ast along the short span

$$M_u x = 0.87 f_y A_{st} x d_{short} \left(1 - \frac{A_{st} f_y}{f_{ck} b d_{short}}\right)$$

b) Spacing = Area of one bar $\times \frac{b}{A_{st, req}}$

Calculation of A_{st} along the long span

$$M_{uy} = 0.87 f_y A_{st} l_{dy} \left(1 - \frac{A_{st} l_{dy}}{A_k b l_{dy}} \right)$$

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{st, req}}$$

Calculation of minimum area of steel

$$\text{min}^m \text{ area} = 0.0015 b D \leq A_{st} \text{ (for mild)}$$

$$= 0.0012 b D \leq A_{st} \text{ (for HYSD)}$$

Note - Curtained bar is 0.1L in each direction and provide 50% of the max steel. At the top or corner of the support. To resist S.M which is 0.1L from the support.

Step-6 Check for shear

The shear is checked on the longer span

$$\text{max}^m \text{ shear force (S.F.)} = \frac{W_u \text{ Left}}{2}$$

$$\text{Nominal shear stress } \tau_v = \frac{V_u}{b l_{dy}}$$

$$P_t = \tau_c = 100 \times \frac{0.15 A_{st} L}{b l_{dy}}$$

$$P_t = \frac{50 A_{st} L}{b l_{dy}}$$

$$\tau_v < \tau_c \text{ (OK)}$$

Step-7 check for development length

$$L_d \leq 1.3 \frac{M_1}{V_u} + L_0$$

$$a) L_0 = 8\phi$$

$$b) L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

Along the longer span

$$M_1 = 0.87 f_y (0.5 A_{st}) l_{dy} \left(1 - \frac{0.5 A_{st} l_{dy}}{A_k b l_{dy}} \right)$$

Along the shorter span

$$a) L_0 = 8\phi$$

$$b) L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

$$M_1 = 0.87 f_y (0.5 A_{st}) l_{dshort} \left(1 - \frac{0.5 A_{st} l_{dshort}}{A_k b l_{dshort}} \right)$$

step-8 check for deflection

$$A.S = 0.58 f_y \times \frac{A_{st req}}{A_{st prov}}$$

check

$$\frac{\text{actual span}}{d} < \frac{\text{allowable span}}{d}$$

step-9 Detailing of two-way slab

$$D_t = 0.1 \cdot 11.12$$

Q. A drawing room of residential building measures $4.3 \times 6.55 \text{ m}$. It is supported on a 350 mm thick wall on all 4-edges. The slab is simply supported at edges with no provision for rest torsion at corners. Design the slab with M20 grade concrete & Fe25 H.V.S.D. reinforcement.

Given data

$$\text{Room size} = 4.3 \times 6.55 \text{ m}$$

$$\text{Live load} = 2 \text{ kN/m}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\frac{L_y}{L_x} = \frac{6.55}{4.3}$$

$$f_y = 415 \text{ N/mm}^2$$

check

$$\frac{L_y}{L_x} = \frac{6.55}{4.3} = 1.52$$

$$\frac{L_y}{L_x} < 2$$

$$D = 180 \text{ mm}$$

$$\phi = 10 \text{ mm}$$

$$C.C = 15 \text{ mm}$$

$$d_{eff} = 180 - 15 - 5 = 160 \text{ mm}$$

step-1 Calculation of T.L

Assume 1 m width

$$b = 1000 \text{ mm}$$

$$D.L = 25 \times 1000 \times 180$$

$$= 4.5 \text{ kN/m}^2$$

$$A.F = 1 \times 2 \text{ kN/m}^2$$

$$T.L = D.L + L.L + A.F$$

$$= 4.5 + 2 + 1$$

$$= 7.5 \text{ m}$$

Factored total load (W_u) = 1.5×7.5

$$= 11.25 \text{ kN/m}$$

Step-2 Calculation of effective span

$$L_{eff} = E.S = L_n + d_{eff}$$

$$= 4.3 + 0.160$$

$$= 4.46 \text{ m}$$

or

L_n + width of support

$$4.3 + 0.350$$

$$= 4.650 \text{ m}$$

which ever is less

$$L_{eff} = 4.5$$

Step-3 Calculation of factored moment

$$M_u = \sigma_y W_u (L_{eff})^2$$

$$= 0.104 \times 11.25 \times (4.5)^2$$

$$= 23.7 \text{ kNm}$$

$$m_y = \sigma_y W_u (L_{eff})^2$$

$$= 0.046 \times 11.25 (L_{eff})^2$$

$$= 0.046 \times 11.25 (4.5)^2$$

$$= 10.47 \text{ kNm}$$

Step-4 Calculation of effective depth required for balanced design

$$m_u = 0.138 f_{ck} b d^2 \quad (\text{for } f_{ck} \leq 415)$$

$$= 0.138 \times 20 \times 1000 \times d^2$$

$$d_{req} = \sqrt{\frac{m_u}{0.138 \times 20 \times 1000}}$$

$$= 92.7 \text{ mm}$$

$$D = d_{req} + e.c + \frac{\phi}{2}$$

$$= 92.7 + 15 + \frac{10}{2}$$

$$= 117.7 \text{ mm}$$

$$= 118 \text{ mm}$$

$$\text{Short span} = D - \frac{\phi}{2} - e.c$$

$$= 118 - \frac{10}{2} - 15$$

$$= 160 \text{ mm}$$

$$\text{long span} = \text{clear} - d = 160 - 10 = 150 \text{ mm}$$

Step 5 Calculation of Ast

a) Ast along the short span

$$m_u x = 0.87 f_y A_{stx} \text{ clear} \left(1 - \frac{A_{stx} f_y}{f_{ck} b d} \right)$$

$$\Rightarrow 23.7 \times 10^6 = 0.87 \times 415 \times A_{stx} \times 160 \left(1 - \frac{A_{stx} \times 415}{20 \times 1000 \times 160} \right)$$

$$\Rightarrow 23.7 \times 10^6 = 57768 A_{stx} \left(1 - \frac{A_{stx} \times 415}{20 \times 1000 \times 160} \right)$$

$$\Rightarrow 7.49 A_{stx}^2 + 57768 A_{stx} - 23.7 \times 10^6 = 0$$

$$\Rightarrow 7.49 A_{stx}^2 + 57768 A_{stx} - 23.7 \times 10^6 = 0$$

$$\Rightarrow 7.49 A_{stx}^2 + 57768 A_{stx} - 23.7 \times 10^6 = 0$$

$$\Rightarrow A_{stx} = 434 \text{ mm}^2$$

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{stx}}$$

$$= \frac{74 \times 10^2 \times 1000}{434}$$

$$= 180 \text{ c/c}$$

Calculation of Asty along long span

$$m_u y = 0.87 f_y A_{sty} \text{ along} \left(1 - \frac{A_{sty} f_y}{f_{ck} b d} \right)$$

$$\Rightarrow 10.47 \times 10^6 = 0.87 \times 415 \times A_{sty} \times 150 \left(1 - \frac{A_{sty} \times 415}{20 \times 1000 \times 150} \right)$$

$$\Rightarrow A_{sty} = 198$$

$$\Rightarrow A_{sty} = 198 \text{ mm}^2$$

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{sty}}$$

$$= \frac{74 \times 10^2 \times 1000}{198}$$

no. of bars

$$n = \frac{A_{stx}}{A_{bar}}$$

$$= \frac{434}{74 \times 10^2}$$

$$= \frac{434}{7400}$$

$$= 5.5 \approx 6$$

$$A_{st \text{ prov}} = 0.6 \times \pi \times 4 \times 8^2 = 198 \text{ mm}^2$$

no. of bar in long span

$$n = \frac{A_{st \text{ new}}}{\pi \times 4 \times 8^2}$$

$$= \frac{198}{\pi \times 4 \times 8^2} = 3.93 \approx 4$$

$$A_{st \text{ prov}} = 4 \times \pi \times 4 \times 8^2 = 201 \text{ mm}^2$$

Calculation of minimum area of steel

$$\text{min}^m \text{ area} = 0.0012 b d < A_{st \text{ by}}$$

$$= 0.0012 \times 1000 \times 180 < A_{st \text{ by}}$$

$$= 216$$

Step-6 check for shear

$$\begin{aligned} \text{max}^m \text{ shear force (S.F.)} &= \frac{W_u \text{ left}}{2} \\ &= \frac{11.25 \times 4.5}{2} \\ &= 25.31 \end{aligned}$$

Nominal shear stress

$$\begin{aligned} \tau_v &= \frac{V_u}{b d \text{ long}} \\ &= \frac{25.31 \times 10^3}{1000 \times 150} \\ &= 0.168 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \tau_c &= 1000 \times \frac{0.5 A_{st}}{b d \text{ long}} \\ &= 1000 \times \frac{0.5 \times 201}{1000 \times 150} \end{aligned}$$

$$= 0.144 \approx 0.145 \text{ N/mm}^2$$

$$k \tau_c = 1.25 \times 0.145$$

$$= 1.13$$

$$\tau_v < k \tau_c$$

$$0.168 < 1.13 \text{ (OK)}$$

Step-7 $m_i = 0.87 f_y A_{st} \text{ along } (1 - \frac{A_{st} f_y}{f_k b d})$

$$= 0.87 \times 415 \times (198) \times 160 \left(1 - \frac{198 \times 415}{20 \times 160 \times 15} \right)$$

$$= 10.5 \text{ MN} \quad 11 \text{ kNm}$$

$$L_d = \frac{\phi 63}{4 \tau_{bd}}$$

$$= \frac{\phi 0.87 \times A_y}{4 \times 1.6}$$

$$= \frac{0.87 \times 415 \phi}{4 \times 1.6} = 56.41 \phi$$

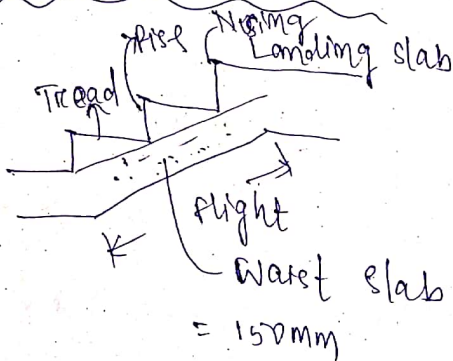
Step-8

10/11-03.11.18

STAIRS

A stair is a member which provides access to the top floors or different floors.

Classification of stairs:-



According to arrangement

- 1) straight stairs
- 2) dog-legged stairs
- 3) open-well stairs

Design steps for stairs:-

Live load = 3 kN/m^2 (for corridors, hospital ward, ball room etc.)

where over crowding is less (live load 2 kN/m^2 for residential building)

→ Live load = 5 kN/m^2 (for over crowding)

Live load on stairs = 2 kN/m^2

Given data

- i) Live load
- ii) Rise & Tread
- iii) height of floor
- iv) Grade of concrete, steel

a) finding number of stairs

→ height of each flight = $\frac{\text{Floor height}}{2}$

ii) No. of Rises = $\frac{\text{Height of each flight}}{\text{Rise}}$

iii) No. of Tread = $\text{No. of Rise} - 1$

iv) Length of stair case = $\text{No. of Tread} \times (\text{No. of tread} \times \text{width of tread} + 2 \times \text{length of slab width})$

The landing width is assume as 1m.

b) Design of bending

Assume thickness of wall slab = 150mm

Assume 1m length of slab

Step-1 Calculation of Total load (T.L.)

$$= L.L + D.L + F.F$$

$$D.L = 25bD \text{ (PCC)}$$

$$F.F = 24bD \text{ (PCC)}$$

$$F.F = 1 \times 1 \text{ (for one floor)} \text{ kN/m}^2$$

$$\text{Factored total load (Wu)} = 1.5 \times T.L.$$

Step-2 Effective span

$$L_{eff} = E.S = \text{C/C distance bet walls or beam}$$

Step-3 Design load ~~structure~~ ^(m.o.p)

i) ~~Design~~ max^m bending moment

$$= \frac{W_u L^2}{8}$$

For Fe415

$$m_y = 0.138 f_{ek} b d^2$$

$$d_{req} = \sqrt{\frac{m_y}{0.138 f_{ek} b}}$$

$$D = c + d + c + \frac{\phi}{2}$$

$$\left\{ \begin{array}{l} D = 146 \\ D_{pro} = 150 \end{array} \right.$$

$$clear = D - c - c - \frac{\phi}{2}$$

Step-4 Ast ?

$$m_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ek} b d} \right)$$

Step-4

Calculation of min^m steel required

$$= 0.0015 b D \text{ (for mild steel)}$$

$$= 0.0012 b D \text{ (for Fe415)}$$

Step-5

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{st, req}}$$

Step-5

check for shear

$$V_u = \frac{W_u L}{2}$$

$$ii) \tau_v = \frac{V_u}{bd}$$

$$iii) \tau_c = \rho_1 = \frac{100 A_{st}}{bd}$$

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$$iv) \tau_v < \tau_c$$

$$\tau_v < k \tau_c$$

Step 6 Development length

$$i) L_d = 8 \phi$$

$$ii) L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

$$iii) M_{u1} = m_1 = 0.87 f_y A_{st} d \left[1 - \frac{A_{st} f_y}{F_{ck} b d} \right]$$

$$L_d < \frac{m_1}{V_u} + L_0$$

For S.S.B

$$L_d < \frac{1.3 m_1}{V_u} + L_0$$

$$\frac{100}{P}$$

Step 7 check for deflection

$$\frac{\text{Allowable span}}{d} < \frac{\text{Basic span}}{d}$$

$$\frac{\text{Basic span}}{d} = 20 \text{ (S.S.B.)}$$

$$F.S.M = \frac{0.58 f_y A_{st} \text{reqd}}{A_{st} \text{prov}}$$

$$\text{Allowable span} = M.F \times 20$$

$$\frac{\text{Actual span}}{d} < \frac{\text{allowable span}}{d} \quad (\text{OK})$$

B Design for flight

Step 1 Calculation of total load

Self weight of waist slab on plan

$$D.L = \text{Thickness of slab} \times 25 \times \sqrt{\frac{R^2 + T^2}{T}}$$

$$\text{Floor finish} = \frac{R + T}{T} \times 1 \text{ kN/m}^2$$

$$\text{Live load} = 3 \text{ kN/m}^2$$

$$\text{Weight of step} = \frac{0 + R \times 25}{2000 \text{ mm}} \text{ kN/mm}$$

$$\text{or } \frac{0 + R \times 25}{2 \text{ m}} \text{ kN/m}$$

$$\text{Total load (T.L)} = D.L + F.F + \text{total weight of steps}$$

$$\text{Factored load (W}_u\text{)} = 1.5 \times \text{T.L}$$

Step-2

Design for flexure

FOOTING

DL-15.11.18

ISOLATED FOOTING

It is the most commonly used for RCC column because of its simplicity and economy.

→ The isolated footing for column may be of pad footing (constant thickness) or slope footing.

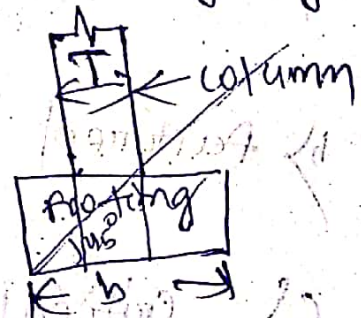
Wall footing:-

Footing for walls may be of two types

a) footing for masonry wall
where the wall is separated from footing.

b) footing for RCC wall
where the wall is cast integrally with the footing.

According to clause no. 24.12
of IS 456:2000



The thickness of the edge of the footing shall not be less than 150mm. For footings on soils, not less than 300mm above the top of piles for footing on piles.

The development length shall be measure from the face of the wall to the end of cantilever minus — clear cover. (footing 50mm).

Step required for evolving and isolated footing.

Step-1 Assume 1m length of footing and extra 10% width of footing

$$b = \frac{\text{Working load} \times 1.1}{ABP}$$

ABP = Allowable bearing pressure on soil

Step-2 Calculate the moment steel

a) Net upward soil pressure = $\frac{\text{Total working load}}{b \times L}$

b) Factored upward soil pressure = $1.5 \times \text{Net upward soil pressure}$

c) Critical section

at the face of the wall = $\frac{b \times L}{2}$

d) To find the depth of the footing

$$d = \sqrt{\frac{M_u}{0.133 f_{ck} b}}$$

Assume overall depth 500mm → 600mm

$$d_{dev} = D - c.c. - \frac{d}{2} \left((\phi = 12, 16, 20) \right) \text{ for footing}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b d} \right)$$

e) To find the spacing

$$\text{Spacing} = \frac{\text{Area of one bar} \times b}{A_{st \text{ req}}}$$

 no. of bars = $\frac{A_{st \text{ req}}}{\text{Area of one bar}}$

f) Distribution steel

$$p_{min} = 0.0012 b D \text{ (gross area)}$$

Step-3 check for shear:

Step-4 Development length

$$L_d = \frac{f_y}{4 \sigma_{bd}} \times \frac{\sigma_{bd}}{f_{ck}} \times \frac{d}{4}$$

$$L_d = \frac{1.1 \times 415 \times 0.87}{4 \times 0.133 \times 25} \times \frac{d}{4}$$

Date - 16.11.18

Design the footing for an R.C.C wall 200mm width and thick and carrying a total working load of 400 kN/m. The A.P. on soil is 200 kN/m². The materials are M20 grade concrete and Fe15.

Given data

Thickness = 200 mm

Total working load = 400 kN/m

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$ Ast

A.P. on Soil = 200 kN/m²

Assume 1m length of footing

$$b = \frac{\text{Working load} \times 1.1}{\text{A.P.}} = \frac{400 \times 1.1}{200} = 2.2 \text{ m}$$

$$= \frac{400 \times 10^3 \times 1.1}{200 \times 10^3} = 2.2 \text{ m}$$

Step-2

Calculate the moment steel

a) Net upward soil pressure = $\frac{\text{total working load}}{b \times l}$

$$= \frac{400 \times 10^3}{2.2 \times 1}$$

$$= 181.81 \text{ kN/m}^2$$

b) factored upward soil pressure

$$W_u = 1.5 \times \text{Net upward soil pressure}$$

$$= 1.5 \times 181.81$$

$$= 272.71 \text{ kN/m}^2$$

c) Critical section at the face of the wall

$$\text{wall} = \frac{b - T}{2}$$

$$= \frac{2.2 - 0.2}{2} = 1 \text{ m}$$

d) To find the depth of the footing

$$d = \sqrt{\frac{m_u}{0.138 f_{ck} b}}$$

$$M_u = \frac{w_u l^2}{2} = \frac{272.71 \times 1^2}{2}$$

$$= 136.36 \text{ kNm}$$

$$d = \sqrt{\frac{136.36}{0.138 \times 20 \times 1000}} = 222 \text{ mm}$$

~~management is essential for~~
~~cuts and schemes.~~

Assume the overall depth 500 mm

$$d_{\text{prov}} = 500 - 50 - \frac{16}{2}$$

$$= 492 \text{ mm}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{f_{ck} b d} \right)$$

$$= 0.87 \times 415 \times A_{st} \times 492 \left(1 - \frac{A_{st} \times 415}{20 \times 492} \right)$$

$$136.36 \times 10^3 = 159584.14 A_{st} - 7.49 A_{st}^2$$

$$\Rightarrow 7.49 A_{st}^2 - 159584.14 A_{st} + 136360 = 0$$

$$A_{st \text{ req}} = 795 \text{ mm}^2$$